

Math 105 — Second Midterm

November 14, 2011

Name: _____

Instructor: _____ Section: _____

1. **Do not open this exam until you are told to do so.**
 2. This exam has 10 pages including this cover. There are 8 problems. Note that the problems are not of equal difficulty, so you may want to skip over and return to a problem on which you are stuck.
 3. Do not separate the pages of this exam. If they do become separated, write your name on every page and point this out to your instructor when you hand in the exam.
 4. Please read the instructions for each individual problem carefully. One of the skills being tested on this exam is your ability to interpret mathematical questions, so instructors will not answer questions about exam problems during the exam.
 5. Show an appropriate amount of work (including appropriate explanation) for each problem, so that graders can see not only your answer but how you obtained it. Include units in your answer where that is appropriate.
 6. You may use any calculator except a TI-92 (or other calculator with a full alphanumeric keypad). However, you must show work for any calculation which we have learned how to do in this course.
 7. If you use graphs or tables to find an answer, be sure to include an explanation and sketch of the graph, and to write out the entries of the table that you use.
 8. **Turn off all cell phones and pagers**, and remove all headphones.
 9. You must use the methods learned in this course to solve all problems.
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Problem	Points	Score
1	14	
2	12	
3	10	
4	13	
5	13	
6	10	
7	14	
8	14	
Total	100	

1. [14 points] No work or explanation is expected on this page.
- a. [4 points] If the graph of the function $y = k(w)$ has a vertical asymptote $w = 5$ and a horizontal asymptote $y = 3$, then what, if any, asymptotes does the graph of $y = 2k(4w) + 3$ have? (Write “None” in the blank if no asymptotes of that type can be found from the information provided.)

Vertical: _____ **Horizontal:** _____

- b. [2 points] If the function $f(q)$ has a zero $q = -2$, find a zero of the function $4f(3(q - 1))$.

Answer: _____

- c. [2 points] Is the function $p(t) = \sin(t) + \pi$ even, odd, or neither? *Circle ONE answer.*

even

odd

neither even nor odd

- d. [2 points] Is there a number whose natural log is 8,675,309? *Circle ONE answer.*

yes

no

- e. [4 points] Suppose $\cos(\phi) = -0.6$ and $\pi \leq \phi \leq 2\pi$. Find $\sin(\phi)$ and $\tan(\phi)$.

Answers: $\sin(\phi) =$ _____ $\tan(\phi) =$ _____

2. [12 points] For each equation below, solve EXACTLY for the specified variable. Show your work carefully and write your final answer on the answer blank provided.

a. [4 points] If $5e^{2t+3} = 6$, solve for t .

Answer: The exact value of t is _____.

b. [4 points] If $\ln(3^x) = \ln(2^x) + 7$, solve for x .

Answer: The exact value of x is _____.

c. [4 points] If $\log(w + 1) - \log(2 - w) = 2$, solve for w .

Answer: The exact value of w is _____.

3. [10 points] Below, you are given a table with some data about two functions, $f(x)$ and $g(x)$:

x	0	1	2
$f(x)$	2	-4	3
$g(x)$	0	5	-1

In addition to this table, we are told the following facts:

- $f(x)$ is an even function.
- $g(x)$ is an odd function.
- There is a third function $h(x)$. The graph of $y = h(x)$ is obtained by shifting the graph of $y = f(x)$ one unit to the right.

Use this information to answer the questions below.

- a. [2 points] Compute $h(1)$.

Answer: $h(1) =$ _____.

- b. [2 points] Compute $f(-1)$.

Answer: $f(-1) =$ _____.

- c. [2 points] Compute $g(2) + g(-2)$.

Answer: $g(2) + g(-2) =$ _____.

- d. [2 points] If $j(x) = f(2x) + 1$, compute the value of $j(1)$.

Answer: $j(1) =$ _____.

- e. [2 points] If $g(x) = k(-2x)$, compute the value of $k(-4)$.

Answer: $k(-4) =$ _____.

4. [13 points] *For each problem on this page, show your work step-by-step. (Don't forget to use appropriate units in your answers.)*

Ozone is a molecule consisting of three oxygen atoms that is unstable and decays to the stable form of oxygen. The half-life of gaseous ozone at a temperature of 20°C is 3 days (72 hours).

- a. [4 points] Find the continuous hourly percent decay rate of gaseous ozone at 20°C .
Give your answer in exact form.

Answer: _____

- b. [4 points] At a temperature of 20°C , how long does it take for the amount of gaseous ozone to be reduced by 90%? *Give your answer in exact form.*

Answer: _____

When ozone is dissolved in water, it is referred to as “aqueous ozone.” At a temperature of 20°C , aqueous ozone decays at a rate of 12.5% per hour.

- c. [5 points] Suppose that in a 20°C lab, the amount of aqueous ozone is initially 5 times the amount of gaseous ozone. When will the two amounts be equal?
Give your answer in exact form.

Answer: _____

5. [13 points] In Western Japan (as in many other places around the world), electrical outlets supply power in the form of alternating current, which means that the voltage changes over time. The voltage goes from a maximum of 141 volts to a minimum of -141 volts and back again, 60 times every second. Let $V(t)$ be the voltage of an electrical outlet in Western Japan, where t is time, in seconds. Assume that $V(t)$ is obtained from the function $\sin(t)$ by performing shifts, stretches and/or reflections, and that at time $t = 0$, the voltage of the outlet is at 141 volts.

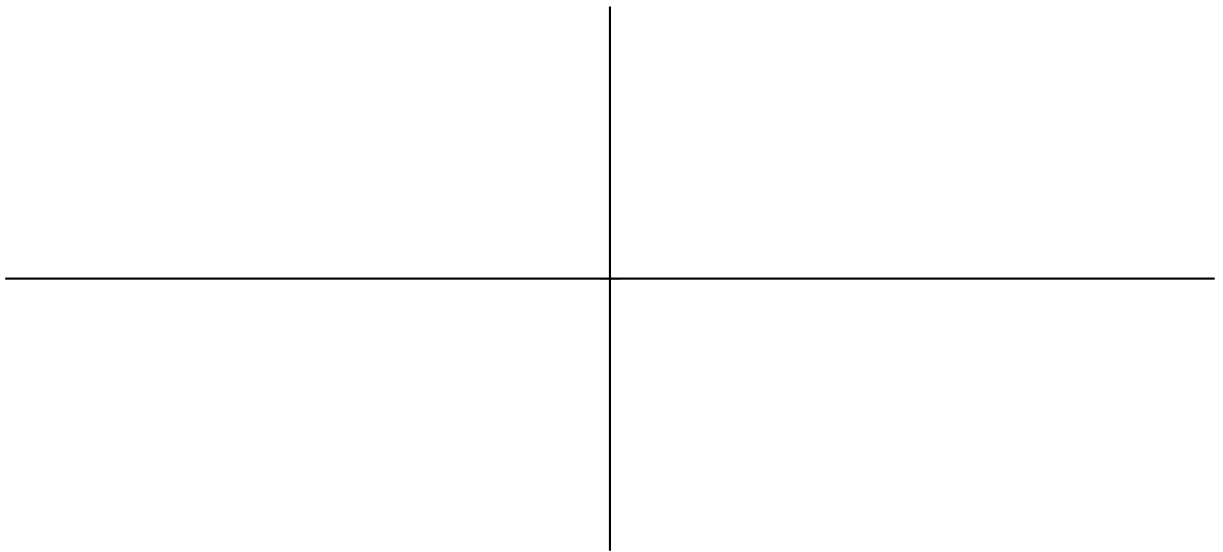
- a. [5 points] Find the period, amplitude, and midline of $V(t)$. (Include units.)

Period: _____

Amplitude: _____

Midline: _____

- b. [5 points] On the axes provided, sketch a graph of $y = V(t)$ for two periods.
(Clearly label the axes and be very careful with the shape and key features of your graph.)



- c. [3 points] The power, $P(t)$ (in Watts) dissipated by a particular electric lightbulb also varies with time. The graph of $y = P(t)$ is obtained from the graph of $y = V(t)$ by performing, in order, the following transformations:
1. A horizontal compression by a factor of $\frac{1}{2}$
 2. A vertical compression by a factor of $\frac{60}{141}$
 3. A shift upward by 60 units
- Find a formula for $P(t)$ in terms of $V(t)$.

Answer: $P(t) =$ _____

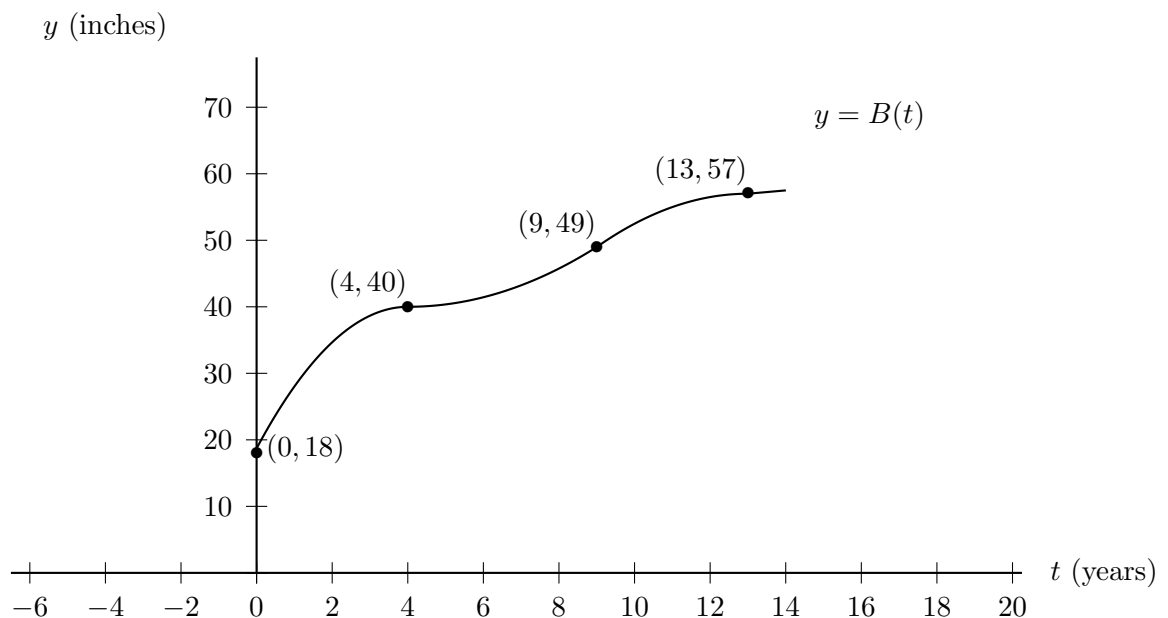
6. [10 points] Mr. and Mrs. Johnson have 4 children: Alana, Brentley, Clarissa, and Donovan. Let $A(t)$, $B(t)$, $C(t)$, and $D(t)$ denote the height, in inches, of Alana, Brentley, Clarissa, and Donovan, respectively, at time t , measured in years since January 1, 1990. Alana was born on January 1, 1990.
- a. [3 points] Alana and Brentley are twins (i.e. they were born at the same time), but Brentley is shorter. He is always 5% shorter than Alana. Write a formula for $B(t)$ in terms of $A(t)$.

Answer: $B(t) =$ _____

- b. [3 points] Clarissa was born exactly 4 years after Alana. Clarissa is always the same height as Alana was when she was the same age. Write a formula for $C(t)$ in terms of $A(t)$.

Answer: $C(t) =$ _____

- c. [4 points] Donovan was born exactly 6 years after Brentley. However, Donovan has a larger build, and is always 4 inches taller than Brentley was at the same age. Below, you are given a portion of the graph of $y = B(t)$. The coordinates of four points on the graph are labeled. Using this information, sketch as much as possible of the graph of $y = D(t)$ on the same axes. **Label four points on your graph.**



7. [14 points] In music, the *pitch* of a tone, P , measured in *cents*, is a function of the tone's frequency, f , measured in Hertz. The pitch is defined to be

$$P = 6000 + k \ln \left(\frac{f}{f_0} \right)$$

where f_0 is the frequency of a tone called "middle C", and k is a constant.

- a. [2 points] What is the pitch of "middle C"? (*Remember to include units.*)

Answer: _____

- b. [3 points] If the frequency of one tone is two times the frequency of middle C, then the pitch of that tone is 7200 cents. Use this information to find the *exact* value of k . Then give an approximation of k rounded to the nearest 0.1.

Exact value of k : _____ **Approximation:** _____

Use the *approximation* of k you found in part (b) to answer the questions below.
(If you were unable to answer part (b), leave your answers below in terms of k .)

- c. [4 points] Let P_1 and P_2 represent the pitches of tones of frequency f_1 and f_2 , respectively. Find a formula for the difference in pitches, $P_2 - P_1$, in terms of the two frequencies f_1 and f_2 . *Simplify your answer; your formula should **not** involve f_0 .*

Answer: $P_2 - P_1 =$ _____

- d. [5 points] The tone called "A above middle C" has a frequency of 440 Hertz. Find the frequency of the tone whose pitch is 400 cents higher than the pitch of "A above middle C." (*Remember to include units.*)

Answer: _____

8. [14 points] There is a bicycle wheel surrounded by a tire of uniform thickness. The wheel itself is 33 centimeters in radius, and the tire is 4 centimeters thick. The wheel has seven evenly-spaced spokes, one of which is initially pointing straight to the right. (See diagram below.)

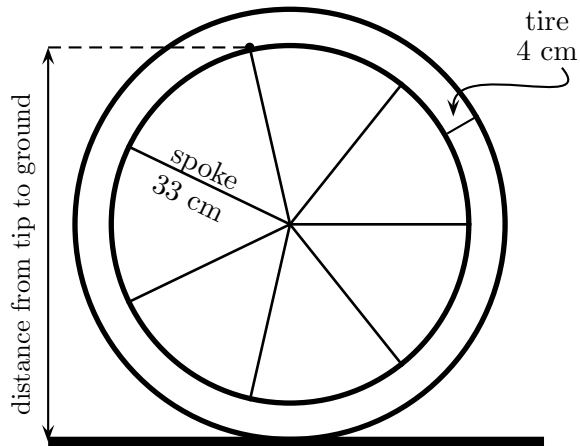


Figure not drawn to scale

- a. [2 points] What is the exact angle (in radians) between two adjacent spokes?

Answer: _____

- b. [4 points] Find the distance from the tip of the highest spoke to the ground.
(This distance is labeled as “distance from tip to ground” in the diagram above.)

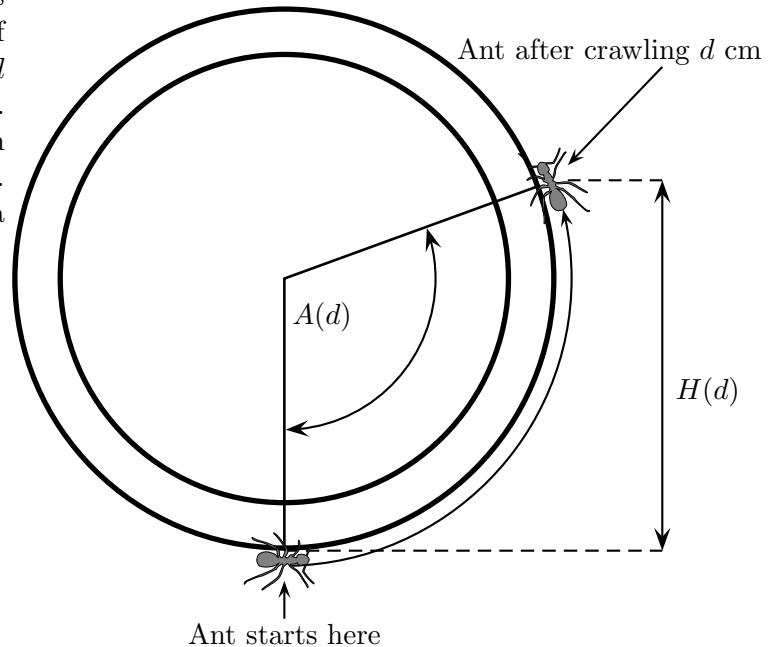
Answer: _____

This problem continues on the next page.

This is a continuation of the problems from the previous page.

Recall: There is a bicycle wheel surrounded by a tire of uniform thickness. The wheel itself is 33 centimeters in radius, and the tire is 4 centimeters thick.

- c. [4 points] One day, while the bicycle is parked, an ant crawls onto the bottom of the tire. The ant crawls for a distance of d centimeters along the outside of the tire. Let $A(d)$ denote the angle, measured in radians, through which the ant crawled. (See diagram on right.) Find a formula for $A(d)$ in terms of d .



Answer: $A(d) =$ _____

- d. [4 points] The ant from part (c), after crawling through a distance of d centimeters, drops off of the tire and falls to the ground. Let $H(d)$ denote the distance, in centimeters, that the ant falls. (See diagram above.) Find a formula for $H(d)$ in terms of d .

Answer: $H(d) =$ _____