

Math 105 — First Midterm

October 8, 2012

Name: _____

Instructor: _____ Section: _____

1. **Do not open this exam until you are told to do so.**
2. This exam has 8 pages including this cover. There are 9 problems. Note that the problems are not of equal difficulty, so you may want to skip over and return to a problem on which you are stuck.
3. Do not separate the pages of this exam. If they do become separated, write your name on every page and point this out to your instructor when you hand in the exam.
4. Please read the instructions for each individual problem carefully. One of the skills being tested on this exam is your ability to interpret mathematical questions, so instructors will not answer questions about exam problems during the exam.
5. Show an appropriate amount of work (including appropriate explanation) for each problem, so that graders can see not only your answer but how you obtained it. Include units in your answer where that is appropriate.
6. You may use any calculator except a TI-92 (or other calculator with a full alphanumeric keypad). However, you must show work for any calculation which we have learned how to do in this course.
7. If you use graphs or tables to find an answer, be sure to include an explanation and sketch of the graph, and to write out the entries of the table that you use.
8. **Turn off all cell phones, smartphones, and other electronic devices**, and remove all headphones.
9. You must use the methods learned in this course to solve all problems.

Problem	Points	Score
1	14	
2	10	
3	4	
4	15	
5	13	
6	5	
7	10	
8	15	
9	14	
Total	100	

1. [14 points] A student named U.M. Student needs to choose a data plan for her new smartphone. She has three plans to choose from.

- The **Maize** Plan costs \$30 per month plus an additional \$10 per gigabyte of data used.
- The **Blue** Plan costs \$15 per month plus an additional \$20 per gigabyte of data used.
- The **Wolverine** Plan costs \$100 per month including unlimited data usage.

- a. [4 points] Let $M(g)$, $B(g)$, and $W(g)$ be the total cost, in dollars, for a month in which she uses g gigabytes of data under the Maize, Blue, and Wolverine Plans, respectively. Find a formula for each of these functions.

$$M(g) = \underline{\hspace{10em}}$$

$$B(g) = \underline{\hspace{10em}}$$

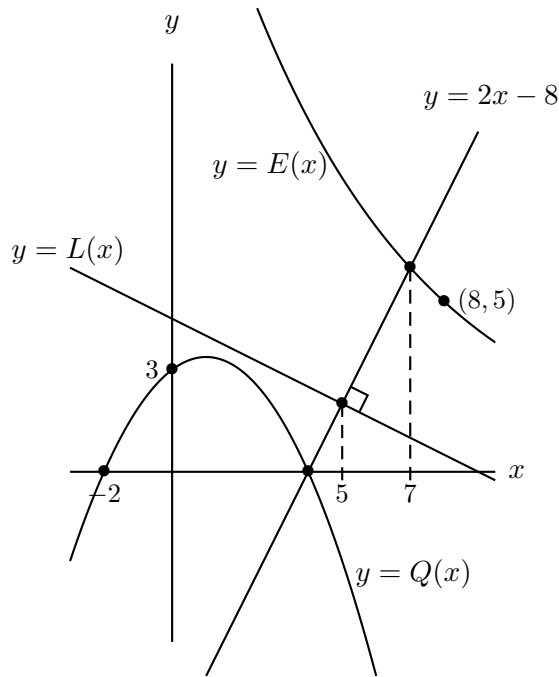
$$W(g) = \underline{\hspace{10em}}$$

- b. [4 points] Sketch the graphs of each of these three functions on the axes below. Be sure to label the axes appropriately, including the values of any intercepts, and clearly indicate which graph is which.



- c. [2 points] If U.M. Student expects to use 1 gigabyte of data per month, which plan would be cheapest? (Justify your answer.)
- d. [4 points] Under what circumstances is the Maize Plan the cheapest? (In other words, for exactly what quantities of monthly data usage is the Maize Plan the cheapest of the three options?) *Show your work and/or explain your reasoning clearly.*

2. [10 points] The graph of $y = 2x - 8$ and of three functions L , Q , and E are shown below. Note that L is linear, Q is quadratic, and E is exponential. Use the information shown in the graph to find formulas for $L(x)$, $Q(x)$, and $E(x)$. *Graphs may not be drawn to scale, so be careful! Use only the information that is labeled in the graph. Show your work clearly and leave all numbers in EXACT FORM.*



Remember: Show your work clearly in the space on this page, and leave all numbers in EXACT FORM. Write your final answers in the answer blanks below.

$$L(x) = \underline{\hspace{10em}}$$

$$Q(x) = \underline{\hspace{10em}}$$

$$E(x) = \underline{\hspace{10em}}$$

3. [4 points] Find the average rate of change of the function $g(t) = 2t^2 - 3t + 4$ between $t = -1$ and $t = -1 + h$. *For full credit, simplify your answer as much as possible.*

4. [15 points] A load of bricks is being lifted by a crane at a constant speed of 5.6 m/s. A brick falls off the stack. The fallen brick's height, in meters above the ground, t seconds after falling off the stack is given by $h(t) = 15.4 + 5.6t - 4.9t^2$.

Throughout this problem, remember to include units and show your work and/or explain your reasoning clearly. (Recall Instruction #7 from the front page.) All answers should be given either in exact form or to at least two decimal places.

- a. [2 points] How high above the ground was the brick when it fell off the stack?

- b. [3 points] How long does it take for the brick to hit the ground?

- c. [3 points] When does the brick reach its highest point?
How high above the ground is the brick at that time?

- d. [3 points] Find the domain and range of the function h in the context of this problem.

Domain: _____ **Range:** _____

- e. [4 points] The supervisor of the construction site sees the brick fall as it passes in front of his office window, which is at a height of 3 meters above the ground. How much time passes between when the supervisor sees the brick and when the brick hits the ground?

5. [13 points] In 1940, there were 6.1 million farms in the United States, and this number decreased by a total of 60% during the next 40 years.
- [2 points] Based on the data above, how many farms were there in the US in 1980?
 - [5 points] Suppose that the number of farms decreased *at a constant rate* from 1940–1980. Find a formula for $F(t)$, the number of millions of farms in the US this model predicts there were t years after 1940 .

According to this model, in what year were there (or will there be) a total of 4 million farms in the US?

- [6 points] Now, suppose instead that the number of farms decreased *at a constant percent rate* from 1940–1980. Under this new assumption, by what percent did the number of farms in the US decrease each year between 1940 and 1980?

Find a formula for $P(t)$, the number of millions of farms in the US this model predicts there were t years after 1940 .

6. [5 points] Let $f(x) = -4x^2 + 12kx - 17$. Use the method of completing the square to rewrite this function in vertex form and then give the coordinates of the vertex.
Show your work step-by-step. Note: Your answers may involve the constant k .

Vertex form: _____

Vertex: _____

7. [10 points] Consider the function q defined by $q(x) = \begin{cases} 3(0.75)^x & \text{if } x \leq -1 \\ 2(x+1)^2 - 8 & \text{if } -1 < x < 2 \end{cases}$

a. [2 points] Evaluate $q(q(0))$.

b. [4 points] Sketch a graph of $y = q(x)$. Carefully label your axes and important points on your graph (including intercepts).

c. [4 points] Find the domain and range of q . (Use either interval notation or inequalities.)

Domain: _____

Range: _____

8. [15 points] The cost of computer memory has changed dramatically over time. Let $C(t)$ be the cost, in dollars per gigabyte, of computer memory t years after 1956. Some estimated data for C is provided in the table below.¹

t	0	33	38	44	48	55
$C(t)$	10,000,000	36,000	1000	20	1	0.035

- a. [3 points] Find and interpret, in the context of this problem, the average rate of change of $C(t)$ for $33 \leq t \leq 38$. (Use a complete sentence and include units.)

- b. [4 points] Based on the data provided in the table above, could the function $C(t)$ be linear, exponential, or neither linear nor exponential? (*Circle one.*)

Linear

Exponential

Neither linear nor exponential

Justify your answer numerically (i.e. show your work and explain your reasoning).

- c. [2 points] Based on the data provided in the table above, is the function $C(t)$ increasing, decreasing, or neither increasing nor decreasing on the entire interval from $t = 0$ to $t = 55$? (*Circle one.*)

Increasing

Decreasing

Neither increasing nor decreasing

- d. [2 points] Based on the data provided in the table above, is the function $C(t)$ concave up, concave down, or neither concave up nor concave down on the entire interval from $t = 0$ to $t = 55$? (*Circle one.*)

Concave Up

Concave Down

Neither concave up nor concave down

- e. [4 points] Estimate $C^{-1}(46)$. Then interpret its meaning in the context of this problem. (Use a complete sentence and include units.)

¹Source: http://en.wikipedia.org/wiki/Memory_storage_density

9. [14 points] A fashion designer has a budget of \$300 for fabric for a fabulous garment. The designer is going to use a combination of denim fabric which costs \$8 per yard and jersey fabric which costs \$12 per yard. (Assume that the fabric store will sell any length of these fabrics, i.e. partial yards are okay.)

Assume that the designer spends the entire budget of \$300 on these two fabrics. Let D be the number of yards of denim and J be the number of yards of jersey that the designer purchases.

- a. [2 points] In one complete sentence, explain why J is a function of D .

Let $f(D)$ be the number of yards of jersey that the designer buys if the designer buys D yards of denim, so $J = f(D)$.

- b. [3 points] Evaluate $f(5)$ and interpret it in the context of this problem.
(Use a complete sentence and include units.)

- c. [3 points] Find a formula for $f(D)$.

- d. [3 points] Find and interpret, in the context of this problem, the D -intercept of the graph of $J = f(D)$. (Use a complete sentence and include units.)

- e. [3 points] Give a practical interpretation of $f^{-1}(k)$ in the context of this problem.
(Use a complete sentence and include units. You do not need to find a formula.)