Math 105 — Second Midterm

November 12, 2012

Name: ______________________________

Instructor: ______________________________ Section: ____________

1. **Do not open this exam until you are told to do so.**
2. This exam has 10 pages including this cover. There are 11 problems. Note that the problems are not of equal difficulty, so you may want to skip over and return to a problem on which you are stuck.
3. Do not separate the pages of this exam. If they do become separated, write your name on every page and point this out to your instructor when you hand in the exam.
4. Please read the instructions for each individual problem carefully. One of the skills being tested on this exam is your ability to interpret mathematical questions, so instructors will not answer questions about exam problems during the exam.
5. Show an appropriate amount of work (including appropriate explanation) for each problem, so that graders can see not only your answer but how you obtained it. Include units in your answer where that is appropriate.
6. You may use any calculator except a TI-92 (or other calculator with a full alphanumeric keypad). However, you must show work for any calculation which we have learned how to do in this course.
7. If you use graphs or tables to find an answer, be sure to include an explanation and sketch of the graph, and to write out the entries of the table that you use.
8. **Turn off all cell phones, smartphones, and other electronic devices**, and remove all headphones.
9. You must use the methods learned in this course to solve all problems.

Problem	Points	Score
1	8	
2	6	
3	5	
4	6	
5	10	
6	14	
7	12	
8	10	
9	11	
10	8	
11	10	
Total	100	

1. [8 points] For each of the statements below, circle "**True**" if the statement is *definitely* true. Otherwise, circle "**False**". You do not need to show any work for this problem.

a. [2 points] The function $g(x) = 3^x + \left(\frac{1}{3}\right)^x$ is an even function.

True False

b. [2 points] The graph of $y = \ln(10x)$ can be obtained from the graph of $y = \ln(x)$ by a vertical shift.

True False

c. [2 points] The line $y = 3$ is a horizontal asymptote of the function $f(x) = e^{10000x} + 3$.

True False

d. [2 points] The function $h(x) = 5\cos(3x)$ is an odd function.

True False

2. [6 points] The graph of the function $g(x)$ contains the point $(-6, 4)$. For each of the functions below, find the coordinates of one point that must be on the graph of the function. Write the coordinates of the point in the form (x, y) on the provided answer blank.

You do not have to show work for this problem.

a. [2 points]

If $h(x) = 0.25g(-0.5x)$, then the graph of $h(x)$ must contain the point ____________________.

b. [2 points]

If $n(x) = g(x+3)-4$, then the graph of $n(x)$ must contain the point ____________________.

c. [2 points]

If $p(x) = -3g(2x-4)$, then the graph of $p(x)$ must contain the point ____________________.

3. [5 points] A colony of bacteria triples in size every 6 days. What is the doubling time of this colony? (Show your work step-by-step, give your final answer in **exact form**, and *include units.*)

Answer: ______________________________

4. [6 points] Let $G(m)$ be the mass (in grams) of the garbage in a dumpster m minutes before 8 am. For each of the functions below, find a formula by applying one or more appropriate transformations to the function G. (*In each case, your final answer should be a formula involving G.*)

a. [2 points] Let $K(m)$ be the mass (in **kilograms**) of the garbage in the dumpster m minutes before 8 am.

Answer: $K(m) =$ ______________________________.

b. [2 points] Let $L(h)$ be the mass (in kilograms) of the garbage in the dumpster h **hours** before 8 am.

Answer: $L(h) =$ ______________________________.

c. [2 points] Let $T(h)$ be the mass (in kilograms) of the garbage in the dumpster h hours before **11 am**.

Answer: $T(h) =$ ______________________________.

5. [10 points] *For potential partial credit, be sure to show your work.*

a. [4 points] Suppose that the domain of $f(t)$ is the interval $[-10, 20)$ and the range of $f(t)$ is the interval $(-8, \infty)$. Find the domain and range of the function $h(t) = 5f(-2t) + 6$.

Domain: ____________________ **Range:** ____________________

b. [3 points] If a weight hanging on a string of length 6 feet swings through 11° on either side of the vertical, how long is the arc through which the weight moves from one high point to the next high point? (*Give your answer in exact form and include units.*)

Answer: ____________________

c. [3 points] The graph of $T(x)$ can be obtained from the graph of $\tan(x)$ by

- first stretching the graph horizontally (away from the vertical axis) by a factor of 3,
- then shifting the graph to the right 5 units,
- then reflecting the graph across the horizontal axis,
- and finally shifting the graph down 2 units.

Find a formula for $T(x)$.

Answer: $T(x) =$ ____________________

6. [14 points] The number of hours of daylight in Ann Arbor varies from a minimum of 9.1 hours of daylight on December 21 to a maximum of 15.3 hours of daylight on June 21 (and then back down to 9.1 hours on the following December 21). Let $L = D(m)$ be the number of hours of daylight in Ann Arbor on a day that is m months after December 21, 2010. Assume that $D(m)$ is a sinusoidal function.

a. [4 points] On the axes provided below, graph *two periods* of the function $L = D(m)$ starting with $m = 0$. (*Clearly label the axes and important points on your graph. Be very careful with the* **shape** *and* **key features** *of your graph.*)

b. [4 points] Find the period, amplitude, and midline of $L = D(m)$. (*Include units* for the period and amplitude.)

Period: ____________________ **Amplitude:** ____________________

Midline: ____________________

c. [4 points] Find a formula for $D(m)$.

Answer: $D(m) =$ ____

d. [2 points] Use your formula from part (c) to estimate the number of hours of daylight in Ann Arbor on April 21. (Show your work and round your answer to the nearest 0.1 hour.)

Answer: ____________________

7. [12 points] Solve each of the equations below. *Show your work step-by-step and write the solutions in* **exact form** *in the answer blanks provided.*

a. [3 points] $5(1.7)^{2y} = 2.4$

Answer: $y =$ ______________________

b. [3 points] $3t - 1 = \log\left(2(10)^{4.6t}\right)$

Answer: $t =$ ______________________

c. [3 points] $e^{\ln(w-4)} = \ln(3.2) - \ln(4)$

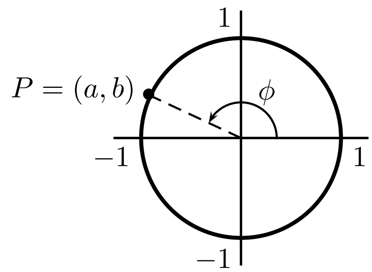
Answer: $w =$ ______________________

d. [3 points] $\log(2p+1) - \log(p-3) = 3$

Answer: $p =$ ______________________

8. [10 points]

The point P (with coordinates (a, b)) is on the unit circle at angle ϕ, as shown in the diagram to the right. Use this information to **find the values below in terms of a and/or b**.
NOTE: Your answers should NOT include function names like "sin", "cos", or "tan".
You do not need to show your work for this problem.



a. [2 points] Find $\sin(\phi)$.

Answer: $\sin(\phi) =$ ____________________

b. [2 points] Find $\tan(-\phi)$.

Answer: $\tan(-\phi) =$ ____________________

c. [2 points] Find $\cos(\phi + \pi)$.

Answer: $\cos(\phi + \pi) =$ ____________________

d. [2 points] Find $\sin(\phi - \frac{\pi}{2})$.

Answer: $\sin(\phi - \frac{\pi}{2}) =$ ____________________

e. [2 points] Find the coordinates of the point at angle ϕ on the circle of radius 7 centered at the point $(-3, 2)$.

Answer: ________________________

9. [11 points] *For this problem, show your work step-by-step and give all answers in* **exact form** *or accurate to at least three decimal places. Include units.*

The concentration (in milligrams per milliliter) of a certain experimental medication ("Medication E") in a patient's bloodstream t hours after injection is $C(t) = De^{-1.5t}$, where D is the concentration immediately after the injection.

a. [2 points] By what percent does the concentration of Medication E in the bloodstream decrease each hour after injection?

Answer: ______________________________

b. [3 points] What is the half-life of Medication E in the bloodstream?

Answer: ______________________________

Suppose that a patient is given two injections (Medications A and B) at the same time.

- Medication A has an initial blood concentration of 3 mg/ml, and its concentration decreases at a *continuous* hourly rate of 25%.
- Medication B has an initial blood concentration of 4.5 mg/ml, and its concentration decreases at a *continuous* hourly rate of 30%.

Let $A(t)$ and $B(t)$ be the blood concentration (in mg/ml) of Medication A and of Medication B, respectively, t hours after the patient receives these injections.

c. [2 points] Find a formula for $A(t)$ and a formula for $B(t)$.

$A(t) =$ ____________________ $B(t) =$ ____________________

d. [4 points] How long after the injections will the concentration of Medication B be only 2% more than the concentration of Medication A in the bloodstream?

Answer: ______________________________

10. [8 points] The management of a new pizza restaurant (called "New Pizza Restaurant" or "NPR") believes that the number of pizzas the restaurant will sell each month is a function of the amount of money it spends on advertising that month. Let $P(A)$ be the average number of pizzas the restaurant expects to sell in a month when it spends A dollars on advertising. Market research suggests that

$$P(A) = 600 + 50\ln(A+1).$$

Throughout this problem, show your work step-by-step and give all answers in **exact form** *or rounded accurately to at least two decimal places. Include units.*

a. [1 point] How many pizzas does NPR expect to sell in a month if they spend no money on advertising?

Answer: ______________________________.

b. [3 points] How much money does NPR plan to spend on advertising in a month if they want to sell 1200 pizzas that month?

Answer: ______________________________.

c. [4 points] A competitor, "OPR", expects to sell an average of $300+45\ln((A+1)^2)$ pizzas in a month when it spends A dollars on advertising. Suppose that in December, OPR and NPR will spend the same amount on advertising and will both expect to sell the same number of pizzas. How much will each restaurant spend on advertising in December?

Answer: ______________________________.

11. [10 points] There is a third pizza restaurant in town known as "TPR". Let $Z(A)$ be the expected number of pizzas TPR will sell in a month when TPR spends A dollars on advertising. Let k be the average amount (in dollars) that TPR spends on advertising each month.

a. [2 points] Write an equation expressing the following statement:
"When TPR spends \$100 more than average on advertising, it expects to sell 25 more pizzas than average."

Answer: ______________________________

For each of the quantities in parts (b)–(e) below, pick the ONE expression from the list of "Answer Choices" that best represents the described quantity. Clearly write the CAPITAL LETTER of your choice on the answer blank provided.

Answer Choices for (b)-(e)

A. $Z(k)$	F. $Z(500k)$	K. $Z(k) - 0.5$	P. $Z^{-1}(k + 0.5)$
B. $Z^{-1}(k)$	G. $1.5Z(k)$	L. $Z(k) + 0.5$	Q. $Z^{-1}(1.5k)$
C. $Z(500)$	H. $0.5Z(k)$	M. $Z(k - 0.5)$	R. $Z^{-1}(1.5Z(k))$
D. $Z^{-1}(500)$	I. $Z(1.5k)$	N. $0.5Z^{-1}(k)$	S. $Z^{-1}(0.5k)$
E. $Z(k + 500)$	J. $Z(0.5k)$	O. $1.5Z^{-1}(k)$	T. $Z^{-1}(0.5Z(k))$

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b. [2 points] Which expression represents TPR's expected monthly pizza sales when \$500 is spent on advertising that month?

Answer: __________

c. [2 points] Which expression represents the amount TPR spends on advertising in a month when it expects to sell 500 pizzas?

Answer: __________

d. [2 points] Which expression represents TPR's expected monthly pizza sales if it spends 50% less on advertising than average?

Answer: __________

e. [2 points] Which expression represents the amount TPR spends on advertising if it expects to sell 50% more pizzas than in a month with average spending on advertising?

Answer: __________