Math 105 — Final Exam

December 14, 2012

Name: ____

Instructor: ______________________________ Section: ______________

1. **Do not open this exam until you are told to do so.**
2. This exam has 11 pages including this cover. There are 13 problems. Note that the problems are not of equal difficulty, so you may want to skip over and return to a problem on which you are stuck.
3. Do not separate the pages of this exam. If they do become separated, write your name on every page and point this out to your instructor when you hand in the exam.
4. Please read the instructions for each individual problem carefully. One of the skills being tested on this exam is your ability to interpret mathematical questions, so instructors will not answer questions about exam problems during the exam.
5. Show an appropriate amount of work (including appropriate explanation) for each problem, so that graders can see not only your answer but how you obtained it. Include units in your answer where that is appropriate.
6. You may use any calculator except a TI-92 (or other calculator with a full alphanumeric keypad). However, you must show work for any calculation which we have learned how to do in this course.
7. If you use graphs or tables to find an answer, be sure to include an explanation and sketch of the graph, and to write out the entries of the table that you use.
8. **Turn off all cell phones, smartphones, and other electronic devices**, and remove all headphones.
9. You must use the methods learned in this course to solve all problems.

Problem	Points	Score
1	5	
2	6	
3	9	
4	12	
5	9	
6	9	
7	8	
8	12	
9	5	
10	4	
11	8	
12	8	
13	5	
Total	100	

1. [5 points] For each of the statements below, circle "**True**" if the statement is *definitely* true. Otherwise, circle "**False**". You do not need to show any work for this problem.

a. [1 point] If a function has more than one zero, then the function is not invertible.

True False

b. [1 point] If $x > 1$, then $100x^{100000} > e^{0.0001x}$.

True False

c. [1 point] If $h(t) = \ln(t)$ then $h^{-1}(t) = \dfrac{1}{\ln(t)}$.

True False

d. [1 point] If a function is concave up, then the function is increasing.

True False

e. [1 point] If $f(x)$ and $g(x)$ are both even functions, then the function $f(g(x))$ is also an even function.

True False

2. [6 points] Solve each of the equations below. *Show your work step-by-step and write the solutions in* **exact form** *in the answer blanks provided.*

a. [3 points] $5e^{2t+7} = 3(4^t)$

Answer: $t =$ ________________________

b. [3 points] $\log(w) + \log(w+3) = 1$

Answer: $w =$ ________________________

3. [9 points] Note that the problems on this page are not related to each other.
(*You do not have to show work. However work shown may be used to award partial credit.*)

a. [3 points] A salesperson at a local department store earns a base salary of \$750 per month plus a commission (bonus) of 8% of her total sales. Let $M(d)$ be the employee's total earnings, in dollars, in a month in which she sells d dollars worth of merchandise. Find a formula for $M(d)$.

Answer: $M(d) =$ ______________________________

b. [3 points] Suppose that the half-life of caffeine in a student's bloodstream is 5 hours. If the student drinks a latte that contains 150 mg of caffeine at 8 am, find a formula for $C(h)$, the amount of caffeine (in milligrams) from that latte that remains in the student's bloodstream h hours after 8 am.

Answer: $C(h) =$ ______________________________

c. [3 points] The monthly revenue of a local business varies seasonally from a low of \$35,000 in February to a high of \$75,000 in August (and back down to \$35,000 the following February). Let $R(t)$ be this company's monthly revenue, in thousands of dollars, t months after January. (Note that $t = 0$ represents January, $t = 1$ represents February, etc.) Assuming that $R(t)$ is a sinusoidal function, find a formula for $R(t)$.

Answer: $R(t) =$ ______________________________

4. [12 points]
Figure 1 below gives some data for an invertible function f and Figure 2 shows the entire graph of a function g. Use this information to answer the questions below.

x	0	1	2	3	4	5	6
$f(x)$	2	6	5	4	1	3	7

Figure 1

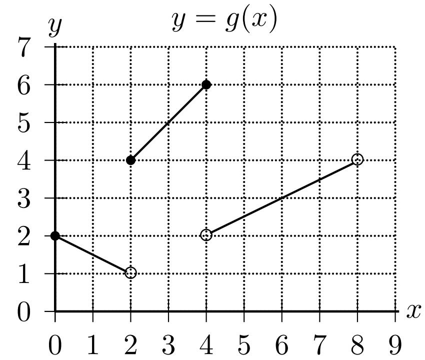

Figure 2

a. [3 points] What is the domain of g? What is the domain of g^{-1}?
(*Use either inequalities or interval notation to give your answers.*)

Domain of g: ________________ **Domain of** g^{-1}: ________________

b. [4 points]

i. Evaluate $3f(2)+1$.

Answer: ____________

ii. Evaluate $g(g(4))$.

Answer: ____________

iii. Evaluate $g(f(1)-1)$.

Answer: ____________

iv. Evaluate $f^{-1}(g^{-1}(3))$.

Answer: ____________

c. [2 points] Find the average rate of change of $f(x)$ between $x = 2$ and $x = 5$.

Answer: ____________

d. [3 points] Suppose $h(x) = 3+4x$. What transformations must be performed on the graph of $y = g(x)$ to obtain the graph of $y = h(g(x))$?
(*Be specific and give the transformations in the appropriate order.)*

5. [9 points] A diver jumps up off of a diving board into a swimming pool below. Until the moment the diver enters the water, his height above the water (measured in feet) t seconds after his feet leave the diving board is $h(t) = -16t^2 + 8t + 10$.
Throughout this problem, remember to show your work and reasoning.
Give your answers in exact form or accurate to at least three decimal places.

a. [3 points] Use the method of completing the square to rewrite the formula for $h(t)$ in vertex form. (*Carefully show your work step-by-step.*)

Answer: $h(t) =$ ________________________

b. [2 points] After how many seconds does the diver reach his maximum height above the pool? What is that maximum height?

After __________ seconds, the diver reaches his maximum height of __________ feet.

c. [2 points] After how many seconds does the diver enter the water?

The diver enters the water __________ seconds after his feet leave the diving board.

d. [2 points] In the context of this problem, what are the domain and range of $h(t)$?
(*Use either inequalities or interval notation to give your answers.*)

Domain: ____________________ **Range:** ____________________

6. [9 points] The tables below provide data from three functions, f, g, and h. Each of these functions is either a *linear* function, an *exponential* function, a *sinusoidal* function, or a *power* function. (Note that there may be either zero, one, or more than one function of each type.)

x	-2	-1	1	2
f(x)	12	1.5	-1.5	-12

x	-3	-1	1	3
g(x)	12	6.5	1	-4.5

x	1	3	5	7
h(x)	32.4	10.8	3.6	1.2

a. [3 points] What type of function is f? (*Circle* ONE *answer.*)

linear exponential sinusoidal power

Find a formula for $f(x)$. (*Show your work carefully and use* exact form.)

Answer: $f(x) =$ ______________________________

b. [3 points] What type of function is g? (*Circle* ONE *answer.*)

linear exponential sinusoidal power

Find a formula for $g(x)$. (*Show your work carefully and use* exact form.)

Answer: $g(x) =$ ______________________________

c. [3 points] What type of function is h? (*Circle* ONE *answer.*)

linear exponential sinusoidal power

Find a formula for $h(x)$. (*Show your work carefully and use* exact form.)

Answer: $h(x) =$ ______________________________

7. [8 points] Consider the polynomials and $a(x) = (x+1)(x^2 - 6x + 3)$ and $b(x) = x(3x^2 + 2)$.

a. [3 points] Find all the zeros of $a(x)$ and of $b(x)$.
(Show your work carefully, and give your answers in *exact form.*)

zero(s) of $a(x)$: ______________________ **zero(s) of $b(x)$:** ______________________

b. [5 points] Let $r(x) = \dfrac{a(x)}{b(x)}$.

Find all intercepts and all horizontal and vertical asymptotes of the graph of $y = r(x)$.
If appropriate, write "NONE" in the answer blank provided.

x-intercept(s): ______________________

y-intercept(s): ______________________

horizontal asymptote(s): ______________________

vertical asymptote(s): ______________________

8. [12 points] Suppose Cato is riding a stationary exercise bicycle. His foot moves a pedal in a circle. Let $h(t)$ be the height (in cm) of the pedal above the ground at time t (in seconds). A formula for $h(t)$ is given by

$$h(t) = 20\sin(2\pi t) + 30.$$

a. [3 points] On the axes provided below, graph *two periods* of the function $P = h(t)$ starting with $t = 0$. (*Clearly label the axes and important points on your graph. Be very careful with the* **shape** *and* **key features** *of your graph.*)

b. [2 points] Find the period and amplitude of $P = h(t)$. (*Include units.*)

Period: ____________________ **Amplitude:** ____________________

c. [4 points] Find all the times t for $0 \leq t \leq 2$ when the pedal is exactly 45 cm above the ground. (*Find at least one answer algebraically. Show your work carefully and check that your answers make sense.*)

Answer(s): ______________________________

d. [3 points] Find the length of the arc through which the pedal travels between $t = 0$ and the time the pedal *first* reaches a height of exactly 45 cm.
(*Show your work and reasoning. It may help to sketch a picture.*)

Answer(s): ______________________________

9. [5 points] Find a formula for *one* polynomial $p(x)$ that satisfies *all* of the following conditions.

- The vertical intercept of the graph of $p(x)$ is 7.
- The graph of $p(x)$ has horizontal intercepts -1, 2, and 3 (and no others).
- $\lim_{x\to\infty} p(x) = -\infty$ and $\lim_{x\to-\infty} p(x) = \infty$.
- The degree of $p(x)$ is at most 6.

Show your work and reasoning carefully. You might find it helpful to first sketch a graph. There may be more than one possible answer, but you should give only one answer.

$p(x) =$ ______________________________

10. [4 points] If $K = G(t) = \dfrac{e^t + 3}{7 + e^t}$ find a formula for $G^{-1}(K)$.

Answer: $G^{-1}(K) =$ ______________________________

11. [8 points] Every morning, a student gets a cup of coffee from a local coffee shop and then sits down to work. Today the coffee was served at a temperature of 185°F. Let $C(t)$ be the temperature, in degrees Fahrenheit, of the cup of coffee t hours after it was poured today, and let $D(t) = C(t) - 70$.

Throughout this problem, show your work carefully and give all answers in exact form or accurate to at least three decimal places.

a. [1 point] Find $D(0)$.

Answer: $D(0) =$ ____________________

b. [2 points] $D(t)$ is an exponential function with a *continuous* hourly decay rate of 80%. Find a formula for $D(t)$ and then find a formula for $C(t)$

$D(t) =$ ____________________ $C(t) =$ ____________________

c. [1 point] By what percent does $D(t)$ decrease each hour?

Answer: ____________________

d. [2 points] By how many degrees did the temperature of the cup of coffee decrease within the first 30 minutes after it was poured?

Answer: ____________________

e. [2 points] Find and interpret, in the context of this problem, any horizontal asymptotes of the function $C(t)$.

12. [8 points] In preparation for an upcoming party, you are deciding where to buy a large supply of candy. You have investigated two sources. Define function C and T as follows.

- It costs $C(p)$ dollars to buy p pounds of candy from the Candy Company.
- For d dollars, you can buy $T(d)$ pounds of candy from Tasty Sweets.

a. [1 point] Write an equation that expresses the fact that it costs $25 to buy 10 pounds of candy from Tasty Sweets.

Answer: ____________________________

b. [1 point] Write an expression that gives the cost of purchasing k pounds of candy from Tasty Sweets.

Answer: ____________________________

c. [2 points] Write an equation that expresses the fact that it costs $10 more to buy 20 pounds of candy from the Candy Company than to buy 15 pounds of candy from the Candy Company.

Answer: ____________________________

d. [2 points] The Candy Company claims that purchasing twice as much candy always costs less than twice as much. Express this statement as an inequality involving C and p.

Answer: ____________________________

e. [2 points] Interpret the meaning of the equation $T(C(15)) = 20$ in the context of this problem. (*Use a complete sentence.*)

---

13. [5 points] (Your score on this problem was determined when you took the LA Post-Test.)