Math 105 — Second Midterm

November 11, 2013

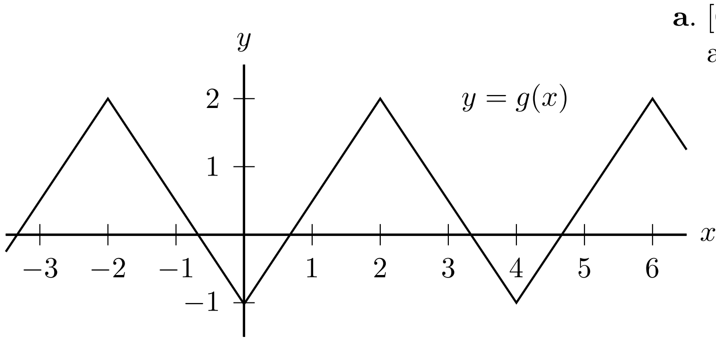
Name: ______________________________

Instructor: ______________________ Section: __________

1. **Do not open this exam until you are told to do so.**
2. This exam has 9 pages including this cover. There are 10 problems. Note that the problems are not of equal difficulty, so you may want to skip over and return to a problem on which you are stuck.
3. Do not separate the pages of this exam. If they do become separated, write your name on every page and point this out to your instructor when you hand in the exam.
4. Please read the instructions for each individual problem carefully. One of the skills being tested on this exam is your ability to interpret mathematical questions, so instructors will not answer questions about exam problems during the exam.
5. Show an appropriate amount of work (including appropriate explanation) for each problem, so that graders can see not only your answer but how you obtained it. Include units in your answer where that is appropriate.
6. You may use any calculator except a TI-92 (or other calculator with a full alphanumeric keypad). However, you must show work for any calculation which we have learned how to do in this course.
7. If you use graphs or tables to find an answer, be sure to include an explanation and sketch of the graph, and to write out the entries of the table that you use.
8. **Turn off all cell phones, smartphones, and other electronic devices**, and remove all headphones.
9. You must use the methods learned in this course to solve all problems.

Problem	Points	Score
1	8	
2	7	
3	12	
4	10	
5	13	
6	11	
7	12	
8	12	
9	5	
10	10	
Total	100	

1. [8 points] *Note: You do not need to show any work for this problem.*
The graph of a periodic function g is shown below.



a. [6 points] Find the period, amplitude, and equation of the midline of $y = g(x)$.

Period: ______________

Amplitude: ______________

Midline: ______________

b. [2 points] Based on the graph shown above, does the function g appear to be even, odd, neither even nor odd, or both even and odd? *Circle the* ONE *best answer.*

○ even ○ odd ○ neither even nor odd ○ both even and odd

2. [7 points] *Note: You do not need to explain your reasoning for this problem.*

Paula Panda loves sleeping and relaxing on a bed of leaves. Let $P(d)$ be the number of leaves in Paula's sleeping area at noon d days after January 1, 2013.

a. [2 points] Paula's younger brother Red copies his big sister. Each day, he counts the number of leaves in Paula's sleeping area and then makes sure that he has that same number of leaves in <u>his</u> sleeping area the very next day. Let $R(d)$ be the number of leaves in Red's sleeping area at noon d days after January 1, 2013.
Find a formula for $R(d)$ in terms of P and d.

Answer: $R(d) =$ ______________

b. [2 points] Paula's cousin Carrie prefers an extra thick layer of leaves. She always makes sure that she has seven more than twice as many leaves as Paula does on the same day. Let $C(d)$ be the number of leaves in Carrie's sleeping area at noon d days after January 1, 2013. Find a formula for $C(d)$ in terms of P and d.

Answer: $C(d) =$ ______________

c. [3 points] Let $L(w)$ be the number of <u>hundreds</u> of leaves in Paula's sleeping area w <u>weeks</u> after January 1, 2013. Find a formula for $L(w)$ in terms of P and w.

Answer: $L(w) =$ ______________

3. [12 points] Cleaver the beaver is building a large dam to protect against predators. After 4 hours of working, the dam he is building is 24 cm high. After 16 hours of working, the dam he is building is 180 cm high. Let $C(t)$ be the height of Cleaver's beaver dam, in cm, after he has been working for t hours. Assume that $C(t)$ is exponential.

a. [4 points] Find a formula for $C(t)$. *You must find your answer algebraically. All numbers in your formula should be in exact form.*

Answer: ______________________

b. [1 point] Find the continuous hourly growth rate of the height of Cleaver's dam. *Round your answer to the nearest* 0.01%.

Answer: ______________________

Cleaver's neighbors, Anne and Barry, are also each building a dam, and they start working at the same time. Let $A(t)$ be the height, in cm, of Anne's dam t hours after she starts working on it, and let $B(t)$ be the height, in cm, of Barry's dam t hours after he starts working on it.

c. [2 points] Write an equation that expresses the following sentence:

"After they have been working for h hours, Anne's dam is 35% taller than Barry's dam."

Note: Your equation may involve A, B, and h.

Answer: ______________________

- Anne's dam starts off 5 cm high, and she builds at a continuous hourly rate of 22%.
- Barry's dam starts off 12 cm high, and he builds at a constant rate of 4 cm per hour.

d. [2 points] Use the information above to find formulas for $A(t)$ and $B(t)$.

Answers: $A(t) =$ ______________ and $B(t) =$ ______________

e. [3 points] When will Anne's dam be 35% taller than Barry's dam? *Round your answer to the nearest* 0.01 *hour. Clearly indicate how you found your solution. (Remember item 7 from the instructions on the front page.)*

Answer: ______________________

4. [10 points] For each of the statements below, circle "**True**" if the statement is *definitely* true. Otherwise, circle "**False**". You do not need to show any work for this problem.

a. [2 points] If $f(x)$ is an even function, then $-f(-x)$ is an odd function.

True False

b. [2 points] If $g(x)$ is periodic with period c, then $g(c) = g(5 + c)$.

True False

c. [2 points] If the continuous annual growth rate of a population is 10%, then the annual growth rate of the population is more than 10%.

True False

d. [2 points] The graph of $y = \log x$ has both a vertical and a horizontal asymptote.

True False

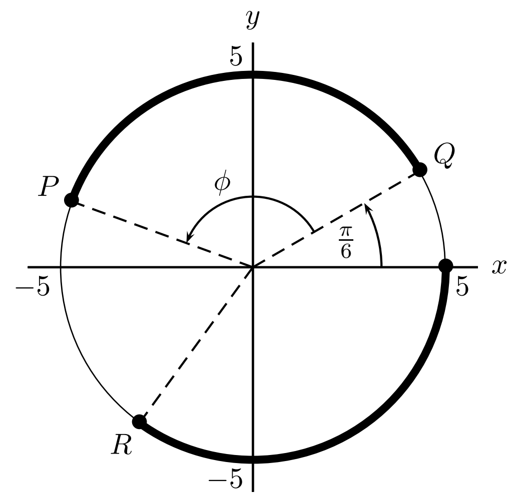
e. [2 points] An angle of one radian is larger than an angle of one degree.

True False

5. [13 points]

The problems on this page refer to the diagram to the right. As shown in the diagram, note the following:

- The points P, Q, and R are on the circle.
- The angle between the positive x-axis and the line segment from the origin to Q is $\frac{\pi}{6}$ radians.
- The angle between the line segment from the origin to Q and the line segment from the origin to P is ϕ radians.



a. [2 points] Find the coordinates of the point Q.
For full credit, each coordinate should be exact and simplified as much as possible.

Answer: The coordinates of Q are (________________ , ________________).

b. [2 points] Find the coordinates of the point P in terms of ϕ.

Answer: The coordinates of P are (________________ , ________________).

c. [2 points] Find the length of the path from Q to P counterclockwise along the circle (the upper path shown in **bold** in the diagram above). Give your answer in terms of ϕ.

Answer: ____________________________

d. [5 points] The length of the counterclockwise path along the circle from the point R to the point $(5, 0)$ (the lower path shown in **bold** in the diagram above) is 11 units. Find the coordinates of the point R. *For full credit, show your work and give decimal approximations rounded to the nearest* 0.01 *unit rather than exact answers.*

Answer: The coordinates of R are (________________ , ________________).

e. [2 points] Based on the diagram above, which of the following statements are true?
Circle ALL *of the statements that are true.*
Circle NONE OF THESE *if none of the statements are true.*

○ $\cos\left(\frac{\pi}{6}\right) > \cos\left(\phi + \frac{\pi}{6}\right)$ ○ $0 > \cos\left(\phi + \frac{\pi}{2}\right)$ ○ NONE OF THESE

○ $\cos(\phi) > \cos\left(\frac{\pi}{6}\right)$ ○ $\sin\left(\phi + \frac{\pi}{6}\right) > 0$

6. [11 points] For each equation below, solve EXACTLY for the specified variable. *Show your work step-by-step and write your answers in* **exact form** *in the answer blanks provided.*

a. [4 points] $12.1e^{0.15p} = 0.78(0.9)^p$

Answer: $p =$ ____________________

b. [4 points] $\dfrac{\ln(z^7) - \ln(z^4)}{\ln(50)} = 5$

Answer: $z =$ ____________________

c. [3 points] $\ln(10e^{-5n}) = 3n + 2$

Answer: $n =$ ____________________

7. [12 points] Last winter, Mollie Mole kept very careful records of her dwindling supply of earthworms. She had 450 grams of earthworms at the beginning of the winter, and 23.5% of her earthworm supply was eaten during the first 10 days of winter.
For this problem, you must find your answers algebraically and show each step carefully.

a. [2 points] *Do not round your answers.*
How many grams of earthworms did Mollie eat during the first 10 days of last winter?

Answer: ______________________

How many grams of earthworms were left in Mollie's supply after the first 10 days of last winter?

Answer: ______________________

Let $W(d)$ be the number of grams of earthworms in Mollie's supply d days after the start of last winter.

b. [4 points] Assuming that Mollie's supply of earthworms decreased exponentially during the first 10 days of last winter, find a formula (in *exact form*) for $W(d)$ for $0 \leq d \leq 10$.

Answer: $W(d) =$ ______________________

c. [1 point] According to your formula above, by what percent did Mollie's supply of earthworms decrease each day during the first 10 days of last winter?

Answer: ______________________

d. [5 points] After the first 10 days, for the rest of last winter, Mollie's remaining supply of earthworms decreased by 6.5% each day. How many total days of winter had passed when her supply dropped below 5 grams? *Remember to find your answer algebraically, showing each step carefully. Then round to the nearest day.*

Answer: ______________________

8. [12 points] *Note that you do not have to show work on this problem. However, any work or reasoning you do show may be considered for partial credit.*

a. [4 points] Suppose h is an <u>odd</u> function and that $(12, -8)$ is a point on the graph of $y = h(t)$. Find the coordinates of <u>two</u> points that must be on the graph of $y = -3h(t+7)$.

Answers: ________________ and ________________

b. [4 points] Suppose the graph of $y = k(x)$ has $y = 4$ as its only horizontal asymptote and $x = -2$ as its only vertical asymptote. If $g(x) = k(-3x) + 11$, what are the *equations* of the horizontal and vertical asymptotes of the graph of $y = g(x)$?

horizontal asymptote: ____________ **vertical asymptote:** ____________

c. [4 points] Suppose the domain of $f(x)$ is the interval $[-4, \infty)$. Find the domain of the function p defined by $p(x) = 5 - f(-2x + 1)$.

Answer: ________________

9. [5 points] An exponentially growing population of mice triples in size every 120 days. How long does it take this population to increase by 400%?
(*Show your work step-by-step, and give your answer in <u>exact form</u>.*)

Answer: ____________________

10. [10 points] Larry the llama and his family of five love having family game night! They find that the more soda they consume on game night, the more board games they can play. Let $b(z)$ be the number of board games they play when the family consumes z ounces of soda. After a few months of family game night, the family finds that

$$b(z) = 3 + 12 \log \left(\frac{z}{p} \right)$$

where p is a positive constant.

a. [3 points] If Larry's family wants to play 13.5 board games, how many ounces of soda should they consume? (Your answer may involve p, but numbers should be in *exact form.*) *Show your work carefully.*

Answer: ______________________________

b. [4 points] *Note: This problem does not depend on part (a) above.*
Suppose the family normally drinks M ounces of soda on game night. How many more board games than usual do they play if they drink 5 times more soda than normal?
Show your work carefully. Your final answer should be a number, i.e. should not include any constants like p or M. Please round to the nearest 0.1 *game.*

Answer: ______________________________

c. [3 points] *Note: This problem does not depend on parts (a) or (b) above.*
Suppose that Larry finds that $b(64) = 5$. Use this to solve *exactly* for p.
Show your work carefully.

Answer: $p =$ ______________________________