Math 105 — Final Exam

December 17, 2013

Name: ______________________________

Instructor: ______________________ Section: __________

1. **Do not open this exam until you are told to do so.**
2. This exam has 11 pages including this cover. There are 12 problems. Note that the problems are not of equal difficulty, so you may want to skip over and return to a problem on which you are stuck.
3. Do not separate the pages of this exam. If they do become separated, write your name on every page and point this out to your instructor when you hand in the exam.
4. Please read the instructions for each individual problem carefully. One of the skills being tested on this exam is your ability to interpret mathematical questions, so instructors will not answer questions about exam problems during the exam.
5. Show an appropriate amount of work (including appropriate explanation) for each problem, so that graders can see not only your answer but how you obtained it. Include units in your answer where that is appropriate.
6. You may use any calculator except a TI-92 (or other calculator with a full alphanumeric keypad). However, you must show work for any calculation which we have learned how to do in this course.
7. If you use graphs or tables to find an answer, be sure to include an explanation and sketch of the graph, and to write out the entries of the table that you use.
8. **Turn off all cell phones, smartphones, and other electronic devices**, and remove all headphones.
9. You must use the methods learned in this course to solve all problems.

Problem	Points	Score
LA Post-Test	5	
1	10	
2	7	
3	10	
4	13	
5	5	
6	5	
7	8	
8	11	
9	5	
10	4	
11	7	
12	10	
Total	100	

1. [10 points] Foghorn is a chicken that is learning how to fly. In fact, he trains every day by jumping off the top of his coop and flapping his wings. Today, his height above the ground, in feet, t seconds after jumping is given by the function $h(t) = -16t^2 + 20t + 6$.
Note that once he lands on the ground, he stays on the ground.

a. [2 points] How long after Foghorn jumps off his coop does he hit the ground?
Be sure to show your work and give your final answer in exact form.

Answer: ______________________

b. [4 points] Use the method of completing the square to put the formula for $h(t)$ into vertex form. *Carefully show your algebraic work step-by-step.*

Answer: $h(t) =$ ______________________

c. [2 points] What is the maximum height Foghorn reaches? **Answer:** ______________

When does he reach his maximum height? **Answer:** ______________

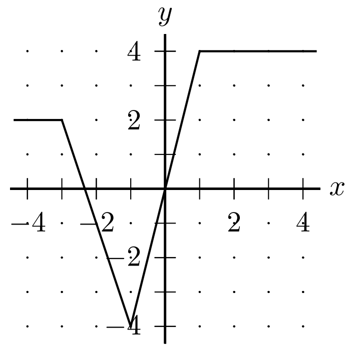
d. [2 points] What are the domain and range of $h(t)$ in the context of this problem?
Use either interval notation or inequalities to give your answers.

Answers: Domain: ______________ Range: ______________

2. [7 points] Invertible functions f and g and a function h are described by the table, formula, and graph below. Use this information to answer the questions that follow.

x	-4	-3	-2	-1	0	1	2	3	4
$f(x)$	3	-2	1	4	-3	0	-4	2	-1

$$g(x) = \begin{cases} 4+x & \text{if } x < 3 \\ 2^x & \text{if } x \geq 3 \end{cases}$$



Graph of $y = h(x)$

Evaluate each of the following quantities, if possible.
If the specified quantity is undefined, write "UNDEFINED".
You do not have to show your work. However, any work you show <u>*may*</u> *be worth partial credit.*

a. [1 point] $f(0)h(-4)$

Answer: ______________

b. [1 point] $3f(g(-2))$

Answer: ______________

c. [1 point] $f^{-1}(h(1)-2)$

Answer: ______________

d. [1 point] $g^{-1}(4)$

Answer: ______________

e. [1 point] $g(g(-1))$

Answer: ______________

f. [1 point] $k(-1)$ if $k(x) = \frac{1}{3}h(3x)$

Answer: ______________

g. [1 point] Find the average rate of change of $h(x)$ between $x = -1$ and $x = 4$.

Answer: ______________

3. [10 points] Let $G(v)$ be the number of minutes it takes Goober the gorilla to eat a meal consisting of v pounds of vegetation.

a. [2 points] Suppose b and n are positive constants.
Give a practical interpretation of the equation $G^{-1}(b) = n$ in the context of this problem.
Use a complete sentence and include units.

b. [4 points] Suppose that there are positive constants c and d so that a formula for $G(v)$ is given by

$$G(v) = cv^d.$$

If $G(2) = 9$ and $G(3) = 18$, find the *exact* values of the constants c and d.

Answers: $c =$ ____________________ and $d =$ ____________________

c. [4 points] Suppose that the number of minutes it takes Goober's friend Toober to eat a meal consisting of v pounds of vegetation is $m = T(v)$, which is given by the formula

$$T(v) = q + \frac{\ln(v+2)}{\ln(5)}$$

for some constant q. Find a formula for $T^{-1}(m)$. *Show your work carefully.*
Note that your answer should be in exact form and be given in terms of m and q.

Answer: $T^{-1}(m) =$ ________________________

4. [13 points] Severus Snake is slithering along the banks of a river. At noon, a scientist starts to track Severus's distance away from the edge of the river. After a few minutes, the scientist realizes Severus's distance away from the edge of the river is a sinusoidal function. Let $D(t)$ be Severus's distance, in centimeters, away from the edge of the river t seconds after noon.

a. [5 points] At noon exactly, the scientist notes that Severus is 97 centimeters away from the edge of the river, which is the farthest away he ever gets. Three seconds after that, Severus's distance is 65 centimeters away from the river, the closest he gets.
Graph $y = D(t)$ for $0 \leq t \leq 12$. (*Clearly label the axes and important points on your graph. Be very careful with the* **shape and key features of your graph.**)

b. [6 points] Find the period, amplitude, equation of the midline, and a formula for the sinusoidal function $D(t)$. (*Include units for the period and amplitude.*)

Period: ____________________

Amplitude: ____________________

Midline: ____________________

Formula: $D(t) =$ ______________________________

c. [2 points] How far away from the river is Severus 11 seconds after noon?
Give your answer accurate to at least two decimal places.

Answer: ______________________________

5. [5 points] Find a formula for *one* polynomial $p(z)$ that satisfies *all* of the following conditions:

- $\lim_{z\to\infty} p(z) = -\infty$ and $\lim_{z\to-\infty} p(z) = -\infty$
- The only zeros of $p(z)$ are $z = -2$, $z = 1$, and $z = 3$.
- The point $(2, -12)$ is on the graph of $p(z)$.
- The degree of $p(z)$ is at most 5.

Show your work and reasoning carefully. You might find it helpful to first sketch a graph. There may be more than one possible answer, but you should give only one answer.

Answer: $p(z) =$ ______________________

6. [5 points] Find all solutions to the equation

$$5\tan\left(2x + \frac{\pi}{2}\right) - 13 = 12$$

for x between 0 and 5. *Show your work carefully and give your answer(s) in exact form.*

Answer: $x =$ ______________________

7. [8 points] Freckles and Comet are cats in the same household. Consider the functions F, C, and D which are defined as follows:

- $F(m)$ is the number of ounces of food that Freckles eats in month m.
- $C(m)$ is the number of ounces of food that Comet eats in month m.
- $D(q)$ is the cost of buying q cans of cat food at a time when there are no sale prices.

Assume that D is invertible.

For each of questions below, circle the ONE best answer from among the options provided. If none of the options are correct, circle NONE OF THESE.

Please note: To receive credit, you must clearly circle your choices. (Circle the **entire** *answer. If there is any ambiguity in your answer, you will not receive credit.)*

a. [1 point] What is the total number of ounces of food that Freckles and Comet eat in month m?

$D(m)+C(m)$ $\quad$ $F(m)+C(m)$ $\quad$ $F(C(m))$ $\quad$ $C(F(m))$ $\quad$ NONE OF THESE

b. [2 points] Suppose that there are 3 ounces of food per can. What is the total cost of the food Freckles eats in month 4?

$\dfrac{D(4)}{3}$ $\quad$ $3D(4)$ $\quad$ $D(3F(4))$ $\quad$ $D\left(\dfrac{F(4)}{3}\right)$ $\quad$ NONE OF THESE

c. [1 point] Let $A(q)$ be the average cost per can of buying q cans of cat food. Which of the following is a formula for $A(q)$?

$D^{-1}(q)$ $\quad$ $\dfrac{q}{D(q)}$ $\quad$ $\dfrac{F(q)+C(q)}{2}$ $\quad$ $\dfrac{D(q)}{q}$ $\quad$ NONE OF THESE

d. [1 point] When there are no sale prices, how many cans of cat food can be purchased at a time for \$20?

$D(20)$ $\quad$ $D^{-1}(20)$ $\quad$ $20D^{-1}(q)$ $\quad$ $\dfrac{1}{D(20)}$ $\quad$ NONE OF THESE

e. [2 points] Suppose that Comet eats at least twice as much food each month as Freckles eats. Which one of the following inequalities most accurately describes this relationship?

$C(m)\le 2F(m)$ $\quad$ $C(m)\ge 2F(m)$ $\quad$ $2C(m)\le F(m)$ $\quad$ $2C(m)\ge F(m)$

f. [1 point] If cat food goes on sale for 40% off its regular price, what is the cost of buying 20 cans of cat food at one time?

$0.6D(20)$ $\quad$ $1.4D(20)$ $\quad$ $0.4D(20)$ $\quad$ $D(8)$ $\quad$ NONE OF THESE

8. [11 points] On the beaches of Mexico, there is a population of picky snails that wait for special shells to wash up onto the shore. These snails can only live in these particular shells, as the snails have become accustomed to the comfort in these shells.
Suppose the number of hundreds of special shells on the beaches of Mexico t years after the beginning of 2013 is

$$h(t) = (t^2 + 7)(4t - 7)^2$$

and the population, in hundreds, of picky snails t years after the beginning of 2013 is

$$p(t) = (t - 2)^2(8t^2 + 30).$$

Throughout this problem, remember to clearly show your work and reasoning.

a. [3 points] Find the leading term and any zeros of $h(t)$. If appropriate, write "NONE" in the answer blank provided.

Answers: Leading Term: ________________ Zero(s): ________________

b. [3 points] The number of shells per snail is $Q(t) = \dfrac{h(t)}{p(t)}$.

Find the equations of all vertical asymptotes ("V.A.") and horizontal asymptotes ("H.A.") of the graph of $y = Q(t)$. If appropriate, write "NONE" in the answer blank provided.

Answers: V.A.: ________________ H.A.: ________________

There is a competitive population of crabs that live on the same beaches. Suppose that there are 1200 of these crabs at the beginning of 2013, and that the population grows at a *continuous annual rate* of 35%. Let $c(t)$ be the population, in hundreds, of these crabs t years after the beginning of 2013.

c. [2 points] Find a formula for $c(t)$.

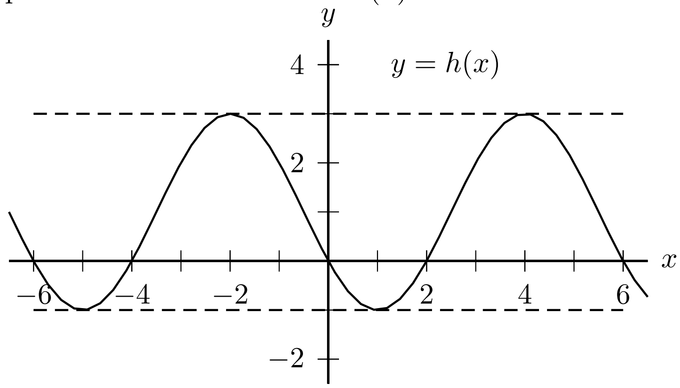
Answer: $c(t) =$ ________________

d. [3 points] The crabs like the same special shells as the snails do. Write a formula for the ratio of the number of shells to the number of crabs t years after the beginning of 2013.

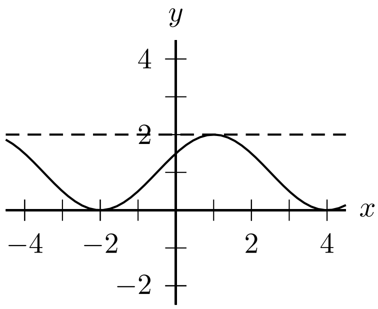
Answer: ________________

In the long run, what happens to the ratio of the number of shells to the number of crabs? In other words, assuming the functions described in this problem continue to be accurate models, what happens to this ratio after many, many years?
You must clearly indicate your reasoning in order to receive any credit for this problem.

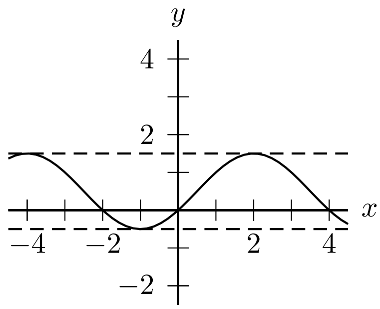
9. [5 points] Note that throughout this problem, you are not required to show your work.
A portion of the graph of a sinusoidal function $h(x)$ is shown below.



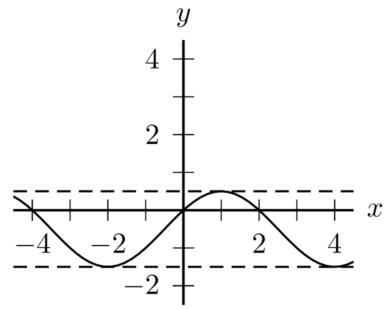
a. [2 points] Which, if any, of the figures below shows part of the graph of $y = -\frac{1}{2}h(x)$?
Note that the scale is smaller than in the original graph above. Be sure to pay attention to the scale indicated on the axes.



(Option A)



(Option B)

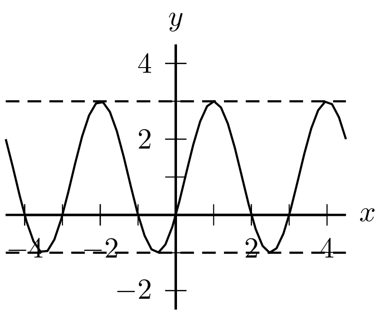


(Option C)

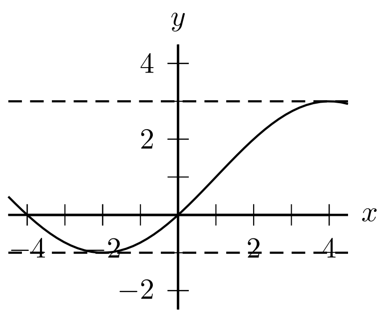
Circle your <u>*one*</u> *final answer below.* (Only the answer you circle below will be graded.)

Option A Option B Option C NONE OF THESE

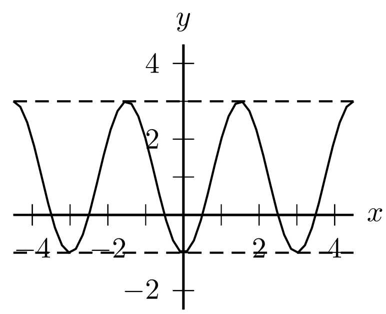
b. [3 points] Which, if any, of the figures below shows part of the graph of $y = h(2x + 2)$?
Note that the scale is smaller than in the original graph above. Be sure to pay attention to the scale indicated on the axes.



(Option A)



(Option B)



(Option C)

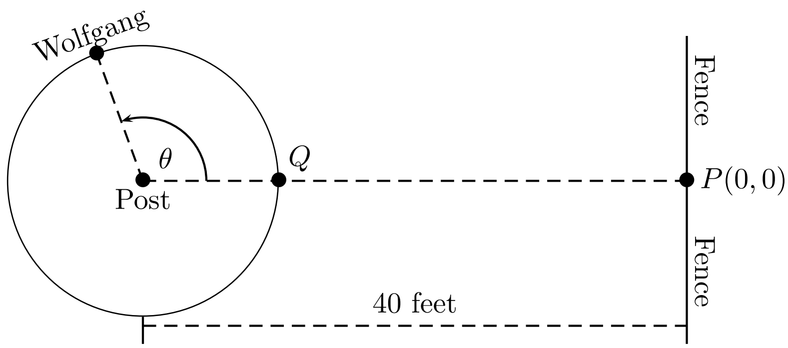
Circle your <u>*one*</u> *final answer below.* (Only the answer you circle below will be graded.)

Option A Option B Option C NONE OF THESE

10. [4 points] Suppose that the number of acorns in Squishy squirrel's nest is proportional to the cube of the number of squirrels currently living there. If there are 113 acorns in his nest when there are two squirrels living there, how many acorns will there be in Squishy's nest when there are four squirrels living there? *Remember to show your work carefully.*

Answer: ______________________

11. [7 points] Wolfgang the wolf is on a 10-foot long leash that is tied to a post that is 40 feet west of a fence.



Because he dislikes being on his leash, he stays 10 feet away from the post at all times.

a. [4 points] Suppose we think of the origin at the point P as shown in the diagram and that the unit of measurement is feet so that the coordinates of the post are $(-40, 0)$. Find Wolfgang's coordinates when he is at the angle θ shown in the diagram. (*Your answer should be in terms of* θ.)

Answer: Wolfgang's coordinates are (____________ , ____________).

b. [3 points] Wolfgang starts walking counterclockwise from the point Q. The angle θ through which Wolfgang has walked is a function of the amount of time he has been walking. Let $\theta = z(t)$ be the angle (in radians) through which Wolfgang has walked after he has been walking for t minutes. Let $A(t)$ be the distance Wolfgang has traveled along the circle in t minutes. Find a function $f(t)$ such that $A(t) = f(z(t))$.

Answer: $f(t) =$ ______________________

12. [10 points] Consider the functions f, g, and h defined as follows:

$$f(x) = a + bx \qquad g(x) = cx^d \qquad h(x) = w(1+r)^x$$

for nonzero constants a, b, c, d, r, and w with $r > -1$.

For each of the questions below, circle all the correct answers from among the choices provided, or circle NONE OF THESE if appropriate.

a. [2 points] The graph of which function(s) definitely has at least one horizontal intercept?

$f(x)$ $g(x)$ $h(x)$ NONE OF THESE

b. [2 points] The graph of which function(s) definitely has at least one horizontal asymptote?

$f(x)$ $g(x)$ $h(x)$ NONE OF THESE

c. [2 points] Which function(s) is(are) definitely invertible?

$f(x)$ $g(x)$ $h(x)$ NONE OF THESE

d. [2 points] How many times could the graph of $f(x)$ intersect the graph of $h(x)$?

0 1 2 3 4 more than 4

e. [2 points] Suppose the graph of h is concave up. Which of the following is(are) definitely true?

$w > 0$ $w < 0$ $r > 0$ $r < 0$ NONE OF THESE