Math 105 — Second Midterm

November 11, 2013

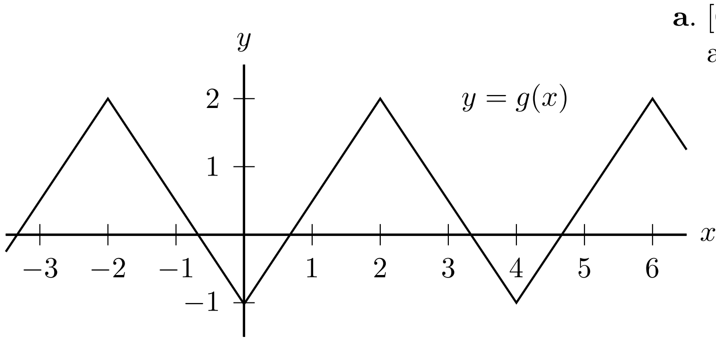
Name: EXAM SOLUTIONS

Instructor: ______________________ Section: __________

1. **Do not open this exam until you are told to do so.**
2. This exam has 9 pages including this cover. There are 10 problems. Note that the problems are not of equal difficulty, so you may want to skip over and return to a problem on which you are stuck.
3. Do not separate the pages of this exam. If they do become separated, write your name on every page and point this out to your instructor when you hand in the exam.
4. Please read the instructions for each individual problem carefully. One of the skills being tested on this exam is your ability to interpret mathematical questions, so instructors will not answer questions about exam problems during the exam.
5. Show an appropriate amount of work (including appropriate explanation) for each problem, so that graders can see not only your answer but how you obtained it. Include units in your answer where that is appropriate.
6. You may use any calculator except a TI-92 (or other calculator with a full alphanumeric keypad). However, you must show work for any calculation which we have learned how to do in this course.
7. If you use graphs or tables to find an answer, be sure to include an explanation and sketch of the graph, and to write out the entries of the table that you use.
8. **Turn off all cell phones, smartphones, and other electronic devices**, and remove all headphones.
9. You must use the methods learned in this course to solve all problems.

Problem	Points	Score
1	8	
2	7	
3	12	
4	10	
5	13	
6	11	
7	12	
8	12	
9	5	
10	10	
Total	100	

1. [8 points] *Note: You do not need to show any work for this problem.*
The graph of a periodic function g is shown below.



a. [6 points] Find the period, amplitude, and equation of the midline of $y = g(x)$.

Period: 4

Amplitude: 1.5

Midline: $y = 0.5$

b. [2 points] Based on the graph shown above, does the function g appear to be even, odd, neither even nor odd, or both even and odd? *Circle the* ONE *best answer.*

∘ $\boxed{\text{even}}$ ∘ odd ∘ neither even nor odd ∘ both even and odd

2. [7 points] *Note: You do not need to explain your reasoning for this problem.*

Paula Panda loves sleeping and relaxing on a bed of leaves. Let $P(d)$ be the number of leaves in Paula's sleeping area at noon d days after January 1, 2013.

a. [2 points] Paula's younger brother Red copies his big sister. Each day, he counts the number of leaves in Paula's sleeping area and then makes sure that he has that same number of leaves in <u>his</u> sleeping area the very next day. Let $R(d)$ be the number of leaves in Red's sleeping area at noon d days after January 1, 2013.
Find a formula for $R(d)$ in terms of P and d.

Solution: On day d, Red will have the number of leaves in his area as Paula had the previous day, which was day $d-1$. So $R(d) = P(d-1)$.

Answer: $R(d) =$ $P(d-1)$

b. [2 points] Paula's cousin Carrie prefers an extra thick layer of leaves. She always makes sure that she has seven more than twice as many leaves as Paula does on the same day. Let $C(d)$ be the number of leaves in Carrie's sleeping area at noon d days after January 1, 2013. Find a formula for $C(d)$ in terms of P and d.

Solution: On day d, Paula has $P(d)$ leaves, so Carrie has $7 + 2P(d)$ leaves.

Answer: $C(d) =$ $7 + 2P(d)$

c. [3 points] Let $L(w)$ be the number of <u>hundreds</u> of leaves in Paula's sleeping area w <u>weeks</u> after January 1, 2013. Find a formula for $L(w)$ in terms of P and w.

Solution: w weeks is equal to $7w$ days, so $P(7w)$ is the number of leaves in Paula's sleeping after w weeks. Dividing this by 100 will give the number of hundreds of leaves.

Answer: $L(w) =$ $\dfrac{P(7w)}{100}$ **or** $0.01P(7w)$

3. [12 points] Cleaver the beaver is building a large dam to protect against predators. After 4 hours of working, the dam he is building is 24 cm high. After 16 hours of working, the dam he is building is 180 cm high. Let $C(t)$ be the height of Cleaver's beaver dam, in cm, after he has been working for t hours. Assume that $C(t)$ is exponential.

a. [4 points] Find a formula for $C(t)$. *You must find your answer algebraically. All numbers in your formula should be in exact form.*

Solution: Since $C(t)$ is exponential, there are constants a and b so that $C(t) = ab^t$. We also have that $C(4) = 24$ and $C(16) = 180$. So, we have $24 = ab^4$ and $180 = ab^{16}$. Taking ratios, we find $\frac{180}{24} = \frac{ab^{16}}{ab^4}$ so $b^{12} = 7.5$ and $b = (7.5)^{1/12}$. We use the equation $ab^4 = 24$ to find the value for a and see that $a\left((7.5)^{1/12}\right)^4 = 24$ so $a = 24(7.5)^{-1/3}$. Thus, $C(t) = 24(7.5)^{-1/3}(7.5)^{t/12} = 24(7.5)^{(t-4)/12}$.

Answer: $C(t) = 24(7.5)^{-1/3}(7.5)^{t/12} = 24(7.5)^{(t-4)/12}$

b. [1 point] Find the continuous hourly growth rate of the height of Cleaver's dam. *Round your answer to the nearest* 0.01%.

Solution: To find the continuous hourly growth rate, we solve the equation $e^k = b$ for k. We have $e^k = (7.5)^{1/12}$ so $k = \frac{1}{12}\ln(7.5) = \frac{\ln 7.5}{12} \approx 16.79\%$.

Answer: 16.79%

Cleaver's neighbors, Anne and Barry, are also each building a dam, and they start working at the same time. Let $A(t)$ be the height, in cm, of Anne's dam t hours after she starts working on it, and let $B(t)$ be the height, in cm, of Barry's dam t hours after he starts working on it.

c. [2 points] Write an equation that expresses the following sentence:

"After they have been working for h hours, Anne's dam is 35% taller than Barry's dam."

Note: Your equation may involve A, B, and h.

Solution: If Anne's dam is 35% is taller than Barry's dam after working for h hours, then $A(h) = 1.35B(h)$.

Answer: $A(h) = 1.35B(h)$

- Anne's dam starts off 5 cm high, and she builds at a continuous hourly rate of 22%.
- Barry's dam starts off 12 cm high, and he builds at a constant rate of 4 cm per hour.

d. [2 points] Use the information above to find formulas for $A(t)$ and $B(t)$.

Solution: If Anne builds at a continuous hourly rate of 22% and her dam starts at 5 cm high, then $A(t) = 5e^{0.22t}$. If Barry builds at a constant rate of 4 cm per hour and his dam starts at 12 cm high, then $B(t) = 12 + 4t$.

Answers: $A(t) =$ $5e^{0.22t}$ and $B(t) =$ $12 + 4t$

e. [3 points] When will Anne's dam be 35% taller than Barry's dam? *Round your answer to the nearest* 0.01 *hour. Clearly indicate how you found your solution. (Remember item 7 from the instructions on the front page.)*

Solution: We want to solve $A(t) = 1.35B(t)$. We graph the functions $A(t)$ and $1.35B(t)$, and using the intersection feature of the calculator, we find that $t \approx -2.46$ or 12.93. Because Anne and Barry started building at time $t = 0$, only the positive solution makes sense in the context of this problem. Therefore, Anne's dam is 35% taller than Barry's dam approximately 12.93 hours after they started to build their dams.

Answer: **About 12.93 hours after starting to build**

4. [10 points] For each of the statements below, circle "**True**" if the statement is *definitely* true. Otherwise, circle "**False**". You do not need to show any work for this problem.

a. [2 points] If $f(x)$ is an even function, then $-f(-x)$ is an odd function.

True $\boxed{\text{False}}$

Solution: In fact, if $f(x)$ is even, then $-f(-x)$ is also even.

To see this graphically, note that the graph of $-f(-x)$ is obtained by reflecting the graph of $f(x)$ across both axes. If the graph of f is symmetric across the vertical axis, then so too is the resulting graph of $-f(-x)$.

We can also check it algebraically. Let $g(x) = -f(-x)$. Then $g(x) = -f(-x) = -f(x)$ since f is even and $g(-x) = -f(-(-x)) = -f(x)$. So $g(-x) = g(x)$ and g is even.

b. [2 points] If $g(x)$ is periodic with period c, then $g(c) = g(5 + c)$.

True $\boxed{\text{False}}$

Solution:
For example, let $g(x) = \sin(x)$. Then the period of g is 2π, but $g(2\pi) \neq g(5 + 2\pi)$.

The following statement would be true:
"If $g(x)$ is periodic with period c and 5 is in the domain of g, then $g(5) = g(5 + c)$."

c. [2 points] If the continuous annual growth rate of a population is 10%, then the annual growth rate of the population is more than 10%.

$\boxed{\text{True}}$ False

Solution: For an exponentially growing function, the continuous growth rate is always less than the percent growth rate. For this specific case, if the continuous annual growth rate is 10%, then the annual growth factor is $e^{0.1} \approx 1.1052$, so the annual percent growth rate is about 10.52%.

d. [2 points] The graph of $y = \log x$ has both a vertical and a horizontal asymptote.

True $\boxed{\text{False}}$

Solution: The graph of $y = \log x$ has vertical asymptote $x = 0$ but does not have a horizontal asymptote.

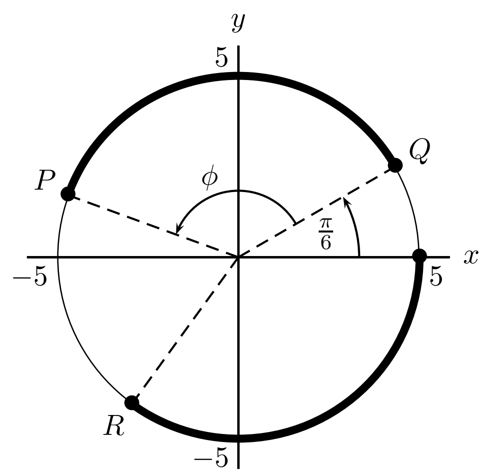
e. [2 points] An angle of one radian is larger than an angle of one degree.

$\boxed{\text{True}}$ False

Solution: An angle of one radian has a measure of $\frac{180}{\pi}^{\circ}$ or about 57.3°.

5. [13 points]
The problems on this page refer to the diagram to the right. As shown in the diagram, note the following:

- The points P, Q, and R are on the circle.
- The angle between the positive x-axis and the line segment from the origin to Q is $\frac{\pi}{6}$ radians.
- The angle between the line segment from the origin to Q and the line segment from the origin to P is ϕ radians.



a. [2 points] Find the coordinates of the point Q.
For full credit, each coordinate should be exact and simplified as much as possible.

Solution: Because the radius is 5 and the angle counterclockwise from the positive x-axis to Q is $\frac{\pi}{6}$, the coordinates of point Q are $(5\cos(\frac{\pi}{6}), 5\sin(\frac{\pi}{6}))$. Recall that $\cos(\frac{\pi}{6}) = \frac{\sqrt{3}}{2}$ and $\sin(\frac{\pi}{6}) = \frac{1}{2}$. Thus the coordinates are $(\frac{5\sqrt{3}}{2}, \frac{5}{2})$.

Answer: The coordinates of Q are $\left(\underline{\quad 5\sqrt{3}/2 \quad}, \underline{\quad 5/2 \quad}\right)$.

b. [2 points] Find the coordinates of the point P in terms of ϕ.

Solution: Because the radius is 5 and the angle counterclockwise from the positive x-axis to P is $\phi + \frac{\pi}{6}$, the coordinates of the point P are $(5\cos(\phi + \frac{\pi}{6}), 5\sin(\phi + \frac{\pi}{6}))$.

Answer: The coordinates of P are $\left(\underline{\quad 5\cos(\phi + \frac{\pi}{6}) \quad}, \underline{\quad 5\sin(\phi + \frac{\pi}{6}) \quad}\right)$.

c. [2 points] Find the length of the path from Q to P counterclockwise along the circle (the upper path shown in **bold** in the diagram above). Give your answer in terms of ϕ.

Solution: Recall that arc length is equal to $r\theta$, where r is the radius of the circle and θ is the angle (measured in radians). In this case, the radius is 5 and the angle is ϕ, so the arc length from Q to P is 5ϕ.

Answer: $\underline{\quad 5\phi \quad}$

d. [5 points] The length of the counterclockwise path along the circle from the point R to the point $(5, 0)$ (the lower path shown in **bold** in the diagram above) is 11 units. Find the coordinates of the point R. *For full credit, show your work and give decimal approximations rounded to the nearest* 0.01 *unit rather than exact answers.*

Solution: First, we find the angle spanned by this arc. The angle satisfies $11 = 5\theta$, so $\theta = 11/5 = 2.2$. Because R is at an angle of θ measured clockwise from the positive x-axis, the coordinates of R are $(5\cos(-2.2), 5\sin(-2.2)) \approx (-2.94, -4.04)$.

Answer: The coordinates of R are $\left(\underline{\quad -2.94 \quad}, \underline{\quad -4.04 \quad}\right)$.

e. [2 points] Based on the diagram above, which of the following statements are true?
Circle ALL *of the statements that are true.*
Circle NONE OF THESE *if none of the statements are true.*

Solution: We use the definitions of sine and cosine to compare the values.

○ $\boxed{\cos\left(\frac{\pi}{6}\right) > \cos\left(\phi + \frac{\pi}{6}\right)}$ ○ $\boxed{0 > \cos\left(\phi + \frac{\pi}{2}\right)}$ ○ NONE OF THESE

○ $\cos(\phi) > \cos\left(\frac{\pi}{6}\right)$ ○ $\boxed{\sin\left(\phi + \frac{\pi}{6}\right) > 0}$

6. [11 points] For each equation below, solve EXACTLY for the specified variable. *Show your work step-by-step and write your answers in* **exact form** *in the answer blanks provided.*

a. [4 points] $12.1e^{0.15p} = 0.78(0.9)^p$

Solution: We take the natural logarithm of both sides of the equation and then apply basic logarithm properties to solve for p.

(Take the natural logarithm of both sides.) $\ln(12.1e^{0.15p}) = \ln(0.78(0.9)^p)$

(Apply basic properties of logarithms.) $\ln(12.1) + \ln(e^{0.15p}) = \ln(0.78) + \ln(0.9^p)$

(Use additional log properties.) $\ln(12.1) + 0.15p = \ln(0.78) + p\ln(0.9)$

(Isolate p on one side of the equation.) $0.15p - p\ln(0.9) = \ln(0.78) - \ln(12.1)$

(Factor out p.) $p(0.15 - \ln(0.9)) = \ln(0.78) - \ln(12.1)$

(Divide to solve for p.) $p = \dfrac{\ln(0.78) - \ln(12.1)}{0.15 - \ln(0.9)}$

Answer: $p = \dfrac{\ln(0.78) - \ln(12.1)}{0.15 - \ln(0.9)} = \dfrac{\ln(0.78/12.1)}{0.15 - \ln(0.9)}$

b. [4 points] $\dfrac{\ln(z^7) - \ln(z^4)}{\ln(50)} = 5$

Solution:

(Multiply both sides of the equation by $\ln(50)$.) $\ln(z^7) - \ln(z^4) = 5\ln(50)$

(Use a basic property of logarithms.) $\ln\left(\dfrac{z^7}{z^4}\right) = 5\ln(50)$

(Simplify.) $\ln(z^3) = 5\ln(50)$

(Use the definition of ln (or exponentiate).) $z^3 = e^{5\ln(50)} = \left(e^{\ln(50)}\right)^5 = 50^5$

(Solve for z.) $z = 50^{5/3}$

Note: One alternate approach is to simplify the left side of the equation as follows: $\ln(z^7) - \ln(z^3) = 7\ln(z) - 4\ln(z) = 3\ln(z)$. Then $\ln(z) = \frac{5\ln(50)}{3}$ and $z = e^{\frac{5\ln(50)}{3}} = 50^{5/3}$.

Answer: $z =$ $50^{5/3}$ **or** $e^{5\ln(50)/3}$

c. [3 points] $\ln(10e^{-5n}) = 3n + 2$

Solution: Using basic properties of logarithms, we can first simplify the left side of the equation as follows:

$$\ln(10e^{-5n}) = \ln(10) + \ln(e^{-5n}) = \ln(10) - 5n.$$

Then we can solve for n.

$$\ln(10) - 5n = 3n + 2$$
$$\ln(10) - 2 = 3n + 5n = 8n$$
$$\frac{\ln(10) - 2}{8} = n$$

Answer: $n = \dfrac{\ln(10) - 2}{8}$

7. [12 points] Last winter, Mollie Mole kept very careful records of her dwindling supply of earthworms. She had 450 grams of earthworms at the beginning of the winter, and 23.5% of her earthworm supply was eaten during the first 10 days of winter.
For this problem, you must find your answers algebraically and show each step carefully.

a. [2 points] *Do not round your answers.*
How many grams of earthworms did Mollie eat during the first 10 days of last winter?

Solution: If she ate 23.5% of her earthworm supply of 450 grams during the first 10 days of last winter, she ate $0.235(450) = 105.75$ grams.

Answer: 105.75 **grams**

How many grams of earthworms were left in Mollie's supply after the first 10 days of last winter?

Solution: She had $450 - 105.75 = 344.25$ grams of earthworms left after the first 10 days of last winter.

Answer: 344.25 **grams**

Let $W(d)$ be the number of grams of earthworms in Mollie's supply d days after the start of last winter.

b. [4 points] Assuming that Mollie's supply of earthworms decreased exponentially during the first 10 days of last winter, find a formula (in *exact form*) for $W(d)$ for $0 \le d \le 10$.

Solution: Since $W(d)$ is exponential and $W(0) = 450$, there is a constant b so that $W(d) = 450\, b^d$. We know that $W(10) = (1 - 0.235)450 = 0.765(450)$, so $0.765(450) = 450b^{10}$. Thus $b^{10} = 0.765$ and $b = 0.765^{1/10} = 0.765^{0.1}$.

Answer: $W(d) =$ $450(0.765)^{d/10}$ **or** $450 \cdot 0.765^{0.1d}$

c. [1 point] According to your formula above, by what percent did Mollie's supply of earthworms decrease each day during the first 10 days of last winter?

Solution: The daily change of her supply is $b - 1 = 0.765^{0.1} - 1 \approx -0.0264 = -2.64\%$, so her supply decreased each day by $1 - 0.765^{0.1} \approx 0.0264 = 2.64\%$.

Answer: $1 - 0.765^{0.1} \approx 2.64\%$

d. [5 points] After the first 10 days, for the rest of last winter, Mollie's remaining supply of earthworms decreased by 6.5% each day. How many total days of winter had passed when her supply dropped below 5 grams? *Remember to find your answer algebraically, showing each step carefully. Then round to the nearest day.*

Solution: At day 10, she has 344.25 grams. If her supply decreases 6.5% each day, then t days after the first 10 days, she has $344.25(0.935)^t$ grams. We solve for t in the equation $344.25(0.935)^t = 5$. We have $0.935^t = \frac{5}{344.25}$, so $\ln(0.935^t) = \ln\left(\frac{5}{344.25}\right)$. Thus

$$t\ln(0.935) = \ln(5) - \ln(344.25) \quad \text{and therefore} \quad t = \frac{\ln(5) - \ln(344.25)}{\ln(0.935)}.$$

So, her supply drops below 5 grams after $10 + \frac{\ln(5)-\ln(344.25)}{\ln(0.935)}$ or about 73 days of winter.
Note: Alternatively, we note that for $d > 10$, a formula for $W(d)$ is given by $W(d) = 344.25(0.935)^{d-10}$. We can then solve for d in $344.25(0.935)^{d-10} = 5$.

Answer: 73 **days**

8. [12 points] *Note that you do not have to show work on this problem. However, any work or reasoning you do show may be considered for partial credit.*

a. [4 points] Suppose h is an odd function and that $(12, -8)$ is a point on the graph of $y = h(t)$. Find the coordinates of two points that must be on the graph of $y = -3h(t+7)$.

Solution: Since $h(t)$ is odd and $(12, -8)$ is a point on the graph of $h(t)$, the point $(-12, 8)$ is also on the graph of $h(t)$. We obtain the graph of $-3h(t + 7)$ from the graph of $h(t)$ by first stretching vertically by a factor of 3, then reflecting across the horizontal axis, and finally shifting left 7 units. This takes the points $(12, -8)$ and $(-12, 8)$ to the points $(5, 24)$ and $(-19, -24)$, respectively.

Answers: $(5, 24)$ and $(-19, -24)$

b. [4 points] Suppose the graph of $y = k(x)$ has $y = 4$ as its only horizontal asymptote and $x = -2$ as its only vertical asymptote. If $g(x) = k(-3x) + 11$, what are the *equations* of the horizontal and vertical asymptotes of the graph of $y = g(x)$?

Solution: The graph of $k(-3x) + 11$ is obtained from the graph of $k(x)$ by compressing horizontally by a factor of $1/3$, then reflecting across the vertical axis, and finally shifting up 11 units. These transformations take the horizontal asymptote $y = 4$ to $y = 15$ and the vertical asymptote $x = -2$ to $x = 2/3$.

horizontal asymptote: $y = 15$ **vertical asymptote:** $x = \frac{2}{3}$

c. [4 points] Suppose the domain of $f(x)$ is the interval $[-4, \infty)$. Find the domain of the function p defined by $p(x) = 5 - f(-2x + 1)$.

Solution: The only transformations involved in obtaining the graph of $y = p(x)$ from the graph of $y-f(x)$ that affect the domain of the function are the horizontal transformations. Because $p(x) = 5 - f(-2x + 1) = 5 - f(-2(x - \frac{1}{2}))$, the horizontal transformations involved are, in order, a horizontal compression by a factor of $1/2$ towards the vertical axis (changing the domain from $[-4, \infty)$ to $[-2, \infty)$), a reflection across the vertical axis (changing the domain from $[-2, \infty)$ to $(-\infty, 2]$), and, finally, a shift to the right by $1/2$ unit (taking the domain to $(-\infty, \frac{5}{2}]$).

Answer: $(-\infty, \frac{5}{2}]$

9. [5 points] An exponentially growing population of mice triples in size every 120 days. How long does it take this population to increase by 400%?
(*Show your work step-by-step, and give your answer in exact form.*)

Solution: Let P_0 be the initial population of mice, and let $P(t)$ be the population of the mice after t days. Then, since the population is growing exponentially there is a growth factor b so that $P(t) = P_0 b^t$. Since the population triples every 120 days, we know that $P(120) = 3P_0$ so $3P_0 = P_0 b^{120}$. Hence $3 = b^{120}$ and $b = 3^{1/120}$. Because we want to find how long it takes for the population to increase by 400%, we want to find the value of t when $P(t) = 5P_0$, i.e. when $5P_0 = P_0(3^{1/120})^t$. We solve for t.

$$5P_0 = P_0 3^{t/120} \quad \text{so} \quad 5 = 3^{t/120} \quad \text{and} \quad \ln(5) = \ln(3^{t/120}) = \frac{t}{120}\ln(3).$$

Thus it takes $t = \dfrac{120\ln(5)}{\ln(3)}$ days for the population to increase by 400%.

Answer: $\dfrac{120\ln(5)}{\ln(3)}$ **days**

10. [10 points] Larry the llama and his family of five love having family game night! They find that the more soda they consume on game night, the more board games they can play. Let $b(z)$ be the number of board games they play when the family consumes z ounces of soda. After a few months of family game night, the family finds that

$$b(z) = 3 + 12\log\left(\frac{z}{p}\right)$$

where p is a positive constant.

a. [3 points] If Larry's family wants to play 13.5 board games, how many ounces of soda should they consume? (Your answer may involve p, but numbers should be in *exact form.*) *Show your work carefully.*

Solution: Since Larry's family wants to play 13.5 board games, we solve for z in the equation $b(z) = 13.5$. We have $3 + 12\log\left(\frac{z}{p}\right) = 13.5$ so $12\log\left(\frac{z}{p}\right) = 10.5$. Since $10.5/12 = 0.875$ we therefore have $\log\left(\frac{z}{p}\right) = 0.875$. By the definition of the logarithm (or "exponentiating"), we thus find $\frac{z}{p} = 10^{0.875}$, so $z = 10^{0.875}p$.

Answer: $10^{0.875}p$ **ounces of soda**

b. [4 points] *Note: This problem does not depend on part (a) above.*
Suppose the family normally drinks M ounces of soda on game night. How many more board games than usual do they play if they drink 5 times more soda than normal?
Show your work carefully. Your final answer should be a number, i.e. should not include any constants like p or M. Please round to the nearest 0.1 game.

Solution: If they drink 5 times more soda than normal, they drink $5M$ ounces of soda. If they drink $5M$ ounces of soda, they play $b(5M) = 3 + 12\log(5M/p)$ board games. Thus, they play $b(5M) - b(M)$ more board games than usual. We use basic properties of logarithms to simplify this expression.

$$\begin{aligned} b(5M) - b(M) &= \left(3 + 12\log\left(\frac{5M}{p}\right)\right) - \left(3 + 12\log\left(\frac{M}{p}\right)\right) \\ &= 12\log\left(\frac{5M}{p}\right) - 12\log\left(\frac{M}{p}\right) = 12\left(\log\left(\frac{5M}{p}\right) - \log\left(\frac{M}{p}\right)\right) \\ &= 12\log\left(\frac{5M/p}{M/p}\right) = 12\log(5) \approx 8.4. \end{aligned}$$

Answer: 8.4 **board games**

c. [3 points] *Note: This problem does not depend on parts (a) or (b) above.*
Suppose that Larry finds that $b(64) = 5$. Use this to solve *exactly* for p.
Show your work carefully.

Solution: If $b(64) = 5$, then we know that $3 + 12\log\left(\frac{64}{p}\right) = 5$ so $12\log\left(\frac{64}{p}\right) = 2$. Then $\log\left(\frac{64}{p}\right) = \frac{1}{6}$, so $\frac{64}{p} = 10^{\frac{1}{6}}$. Solving for p we find that $p = 64(10^{-\frac{1}{6}})$.

Answer: $p =$ $64(10^{-\frac{1}{6}})$