1. Do not open this exam until you are told to do so.
2. This exam has 13 pages (not including this cover page) and there are 11 problems in total.
3. Turn off and put away all cell phones, pagers, headphones, smartwatches and any other unauthorized electronic devices.
4. Note that the problems are not of equal difficulty, so you may want to skip over a problem if you get stuck and return to it later.
5. Do not separate the pages of this exam. If they do become separated, write your UMID on every page and notify your instructor when you hand in the exam.
6. Please read the instructions for each individual problem carefully. One of the skills being tested on this exam is your ability to interpret mathematical questions, so instructors will not answer questions about exam problems during the exam.
7. You must show an appropriate amount of work (including appropriate explanation) for each problem unless indicated otherwise: the graders will be looking not only for your answer, but also how you obtained it. Include units in your answer where appropriate.
8. You may use a TI-84, TI-89, TI-Nspire or other approved calculator. However, you must show work for any calculation which we have learned how to do in this course. You are not allowed a notecard for this exam.
9. If you use a graph that wasn’t provided to you in answering a problem, be sure to include an explanation and sketch of the graph. If your solution to a problem makes reference to a table, it should be clear which entries of the table you’re using.
10. You must solve the problems on this exam using the methods you have learned in this course.

<table>
<thead>
<tr>
<th>Problem</th>
<th>Points</th>
<th>Score</th>
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<td><strong>Total</strong></td>
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I recognize that there are severe penalties for academic dishonesty and I affirm that I have done nothing to compromise the integrity of this exam, nor do I intend to help anyone else do so.

Initials: __________
1. [8 points] Radioactive waste has been draining into a lake, causing trees in the surrounding forest to wilt and die. On March 24, 2010, local scientists surveyed the surrounding forest, determining that \( S(m) \) thousand trees had begun to wilt \( m \) kilometers out from the center of the lake. The scientists determined that 20 thousand trees were decaying 20 kilometers out, and 5 thousand trees were decaying 60 kilometers out.

**a.** [3 points] Assume that the function \( S \) is invertible. Find the average rate of change of the inverse function \( S^{-1} \) on the interval \([5, 20]\). Write your final answer in the space provided, and include units.

\[
\text{The average rate of change of } S^{-1} \text{ on the interval } [5, 20] \text{ is } \underline{\text{ }}
\]

**b.** [5 points] Find a formula for \( S(m) \) in terms of \( m \), assuming that \( S \) is a power function of \( m \). Your answer should be exact, and you must show your work carefully, writing your final answer in the space provided.

\[
S(m) = \underline{\text{ }}
\]
2. [7 points] Olga runs a factory that produces pitch, and finds that the cost $C$ (in thousands of dollars) to produce $g$ gallons of pitch is given by $C = f(g)$, where:

$$f(g) = 5 + \log (3 + e^{7g})$$

for $g \geq 0$. Note that $f$ is an invertible function.

a. [5 points] Find a formula for the quantity of pitch $f^{-1}(C)$ (in gallons) that the factory must have produced in terms of the total cost $C$ (in thousands of dollars) incurred. You must show your work carefully for this part.

$$f^{-1}(C) = \text{________________________}$$

b. [2 points] What is the range of $f^{-1}(C)$? Write your final answer in the space provided, using inequalities.

The range of $f^{-1}(C)$ is \text{________________________}
3. [8 points] You do not need to show any work for this problem.
   a. [2 points] Which of the following functions dominates all the others as $x \to \infty$? **Circle exactly one of the options below.**
      
      $f(x) = 0.01(1.3)^x$  
      $g(x) = 100x^{10}$  
      $h(x) = 300(0.25)^x$  
      $i(x) = 4^{-2x}$  
      $j(x) = 300 \ln(4|x|)$  
      $k(x) = 100 \left( \frac{6}{5} \right)^x$

   b. [2 points] Which of the following functions dominates all the others as $x \to -\infty$? **Circle exactly one of the options below.**
      
      $f(x) = 0.01(1.3)^x$  
      $g(x) = 100x^{10}$  
      $h(x) = 300(0.25)^x$  
      $i(x) = 4^{-2x}$  
      $j(x) = 300 \ln(4|x|)$  
      $k(x) = 100 \left( \frac{6}{5} \right)^x$

   c. [2 points] Let $f(x)$ be an odd function with:
      
      $\lim_{x \to -3^+} f(x) = -\infty$  \hspace{1cm} and \hspace{1cm} $\lim_{x \to -3^-} f(x) = \infty$

      Suppose that $f(3) = 0$. Evaluate $\lim_{x \to -3^-} f(x)$. Write your answer **in the space provided.** If there is not enough information to evaluate the limit, write **NOT ENOUGH INFORMATION**.

      $\lim_{x \to -3^-} f(x) =$

   d. [2 points] Consider the functions:
      
      $f(x) = 1 + \sqrt{1 + x}$  
      $g(x) = 1 + x$

      Find the formula of a function $h(x)$ for which $f(x) = g(h(x))$. Write your answer **in the space provided.**

      $h(x) =$
4. [8 points] In this problem, you should show your work. All your answers should be exact, and must be found algebraically. Write your final answers in the spaces provided.

For parts (a) and (b), consider the function

\[ F(x) = \frac{(100x^2 + 3)(x^2 + 2x - 1)}{(x^2 - 2x - 3)(2x^2 + 4)} \]

a. [2 points] Find the horizontal intercept(s) of \( y = F(x) \). If the function has no horizontal intercepts, write NONE in the space provided.

\[ \text{Horizontal intercept(s): } \]

b. [2 points] Find the equation(s) of the horizontal asymptote(s) of \( y = F(x) \). If the function has no horizontal asymptotes, write NONE in the space provided.

\[ \text{Horizontal asymptote(s): } \]

c. [4 points] Consider the function

\[ G(x) = \frac{x^2(x^2 + 5)^3}{(x - 2)(x^2 + 5)^4x} \]

Find the equation(s) of the vertical asymptote(s) of \( y = G(x) \), and the x-coordinate(s) of the hole(s) of \( y = G(x) \). If the function has no vertical asymptotes or has no holes, write NONE in the relevant space.

\[ \text{Vertical asymptote(s): } \]

\[ \text{Hole(s): } \]
5. [12 points] Consider the following expressions:

\[
\begin{align*}
9e^{-2x} & \quad 4x + 9 & \quad 9\tan\left(\frac{\pi}{2}x\right) & \quad \frac{18}{1 + e^{-x}} \\
3x^2 + 9 & \quad 9 + \sin\left(\frac{x}{30}\right) & \quad 9\tan(3\pi x) & \quad \ln(2x - 1)
\end{align*}
\]

In each of the following parts, write down in the space provided all of the expressions above which could be formulas for the function described. If more than one expression applies, write them all on the same line and ensure that they are clearly separated. If none of the expressions apply, write NONE OF THE ABOVE.

a. [3 points] The function \(f(x)\) satisfies \(f(x) \geq 9\) for \(x \geq 0\).

\(f(x)\) could be: 

b. [3 points] The function \(g(x)\) is periodic with period \(\frac{1}{3}\).

\(g(x)\) could be: 

c. [3 points] The function \(h(x)\) has a vertical asymptote at \(x = \frac{1}{2}\).

\(h(x)\) could be: 

d. [3 points] The function \(i(x)\) has \(\lim_{x \to \infty} i(x) = C\) where \(C \geq 0\) is a nonnegative constant.

\(i(x)\) could be: 

6. [11 points] For this problem, your final answers must be exact and should be written in the spaces provided.

a. [5 points] Let \( V(t) \) be the voltage across a resistor in a circuit (measured in volts) \( t \) minutes after 8:00 a.m. on January 29, 2013. The function \( V(t) \) is periodic, and it takes 5 minutes to go from a minimum of \(-10\) volts to a maximum of 40 volts. At 8:37 a.m., the voltage across the resistor is \(-10\) volts. Find a formula for \( V(t) \), assuming \( V(t) \) is a sinusoidal function of \( t \).

\[
V(t) = \text{______________________________}
\]

b. [6 points] Find all values of \( t \) in the interval \(-0.5 \leq t \leq 1\) for which:

\[
5 \sin \left(2\pi \left(t + \frac{1}{4}\right)\right) + 3 = 0
\]

Your answer must be found algebraically and should be exact. You must show your work carefully to receive full credit.

\[
The\ solutions\ in\ -0.5 \leq t \leq 1\ are\ \text{______________________________}
\]
7. [8 points] Some values of the function \( f(x) \) are given in the table below.

<table>
<thead>
<tr>
<th>( x )</th>
<th>(-2)</th>
<th>0</th>
<th>2</th>
<th>5</th>
<th>8</th>
<th>10</th>
<th>15</th>
<th>18</th>
<th>20</th>
<th>21</th>
</tr>
</thead>
<tbody>
<tr>
<td>( f(x) )</td>
<td>54.22</td>
<td>30.50</td>
<td>17.16</td>
<td>7.24</td>
<td>13.84</td>
<td>18.24</td>
<td>29.24</td>
<td>5</td>
<td>(-9)</td>
<td>(-20)</td>
</tr>
</tbody>
</table>

Note that all values in the table have been rounded to two decimal places. You must show your work for each part of this problem, and write your final answers in the spaces provided.

a. [3 points] Find a formula for \( f(x) \) valid for \(-2 \leq x \leq 5\), assuming that \( f \) is exponential on the interval \([-2, 5]\).

\[ f(x) = \text{exponential function} \]

\[ f(x) = \text{exponential function} \]

b. [2 points] Find a formula for \( f(x) \) valid for \( 5 \leq x \leq 15\), assuming that \( f(x) \) is linear on the interval \([5, 15]\).

\[ f(x) = \text{linear function} \]

\[ f(x) = \text{linear function} \]

c. [3 points] Show that \( f(x) \) cannot be concave down on the interval \([15, 21]\). Make sure any relevant calculations are clearly shown, and write a brief sentence explaining your reasoning.

<table>
<thead>
<tr>
<th>Calculations</th>
<th>Reasoning</th>
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<tbody>
<tr>
<td>[ ]</td>
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</table>
8. [15 points] The number of hemlock trees in the southern Appalachian mountains is declining as a result of an infestation of hemlock woolly adelgids (a kind of insect).

- There are \( H(d) \) healthy hemlock trees in the southern Appalachian mountains \( d \) days after January 1, 2013.
- There are \( I(d) \) infested hemlock trees in the southern Appalachian mountains \( d \) days after January 1, 2013.

Note that all hemlock trees are considered healthy unless they are infested. Be sure to write your final answers in the spaces provided.

a. [2 points] Let \( J(w) \) be the number of healthy hemlock trees in the southern Appalachian mountains \( w \) weeks after January 1, 2013. Find a formula for \( J(w) \) in terms of any or all of the functions \( H \) and \( I \).

\[
J(w) = \text{______________________________}
\]

b. [3 points] Let \( F(d) \) be the fraction of the hemlock trees in the southern Appalachian mountains that are infested \( d \) days after January 1, 2013. Find a formula for \( F(d) \) in terms of any or all of the functions \( H \) and \( I \).

\[
F(d) = \text{______________________________}
\]

c. [4 points] Let \( K(d) \) be the total number of hemlock trees in the southern Appalachian mountains, in thousands, \( d \) days after January 1, 2013. Find a formula for \( K(d) \) in terms of any or all of the functions \( H \) and \( I \).

\[
K(d) = \text{______________________________}
\]
d. [3 points] The number of hemlock trees \( I \) that are *infested* in the southern Appalachian mountains is *inversely proportional* to the cube of the total amount of money \( M \) (in millions of dollars) that the government spends combating the spread of the adelgids. Write a formula for \( I \) in terms of \( M \), assuming that there were 2,000 infested trees when the government had spent 3 million dollars. You must **show your work** for this part.

\[
I = \frac{2000}{M^3}
\]

e. [3 points] The number of hemlock woolly adelgids \( A \) (in millions) is also a function of the amount of money \( M \) (in millions of dollars) that the government spends to try to preserve the hemlock trees, and is given by:

\[
A(M) = \frac{4}{M}
\]

for \( M \geq 4 \). Find the equation of the horizontal asymptote of \( y = A(M) \), and interpret this horizontal asymptote in practical terms.

*The equation of the horizontal asymptote is ____________________________
9. [7 points] Consider the circle of radius $R$ centered at the point $O$, illustrated below. The diagram is not drawn at scale.

![Diagram of a circle with points A, B, C, D, and O. Line AB contains point O, angles ACB and ADC both have measure $\frac{\pi}{2}$ radians. $\alpha$ is the positive measure of the angle COD (see the diagram), while $L$ is the length of the line segment AC.]

Note that the line $AB$ contains the point $O$, and the angles $ACB$ and $ADC$ both have measure $\frac{\pi}{2}$ radians. $\alpha$ is the positive measure of the angle $COD$ (see the diagram), while $L$ is the length of the line segment $AC$.

a. [2 points] Find the length of the line segment $CD$. Your answer for this part may involve any or all of the constants $R$, $L$ and $\alpha$.

The length of $CD$ is _________________

b. [3 points] Find the (positive) measure of the angle $OAC$ in radians. Your answer for this part may involve the constants $R$ and $L$, but must not include the constant $\alpha$.

The measure of $OAC$ is = ___________________

c. [2 points] Find the length of the (bolded) circular arc $AC$. Your answer for this part may involve any or all of the constants $R$, $L$ and $\alpha$.

The length of the arc $AC$ is = ___________________
10. [11 points] Consider the graphs of \( y = k(x) \) and \( y = \ell(x) \) given below:

You must **show your work** in both parts of this problem to receive full credit. Write your final answers in the spaces provided.

a. [5 points] Find a formula for \( k(x) \), assuming \( k(x) \) is a polynomial of degree seven with zeros at \( x = -1 \), \( x = 0 \) and \( x = 3 \).

\[
k(x) =
\]

b. [6 points] Find a piecewise-defined formula for \( \ell(x) \) on \([-2, 6]\), given that the graph of \( y = \ell(x) \) is made up of a line and a parabola.

\[
\ell(x) = \begin{cases} 
\hline 
\text{ if } & \\
\text{ if } 
\end{cases}
\]
11. [5 points] A portion of the graphs of $y = f(x)$ and $y = g(x)$ are given below. You do not need to show any work for this problem.

![Graphs of f(x) and g(x)]

a. [2 points] Assume that $g(x)$ is an invertible function. Which of the following could be the graph of $y = g^{-1}(x)$? Circle exactly one of the four graphs below.

![Graphs of g^{-1}(x)]

This problem continues on the next page
The graphs of \( y = f(x) \) and \( y = g(x) \) from the previous page have been reproduced below for your convenience.

b. [3 points] Which of the following could be the graph of \( y = g(f(x)) \)? **Circle exactly one of the four graphs below.**