Math 105 — Second Midterm

November 14, 2017

UMID: EXAM SOLUTIONS Initials: ________

Instructor: ________ Section: ________

---

1. **Do not open this exam until you are told to do so.**
2. **Do not write your name anywhere on this exam.**
3. This exam has 11 pages including this cover. There are 10 problems. Note that the problems are not of equal difficulty, so you may want to skip over and return to a problem on which you are stuck.
4. Do not separate the pages of this exam. If they do become separated, write your UMID on every page and point this out to your instructor when you hand in the exam.
5. Please read the instructions for each individual problem carefully. One of the skills being tested on this exam is your ability to interpret mathematical questions, so instructors will not answer questions about exam problems during the exam.
6. Show an appropriate amount of work (including appropriate explanation) for each problem so that graders can see not only your answer, but also how you obtained it. Include units in your answer where that is appropriate.
7. You may use a TI-84, TI-89, TI-Nspire or other approved calculator. However, you must show work for any calculation which we have learned how to do in this course.
8. If you use graphs or tables to find an answer, be sure to include an explanation and sketch of the graph, and to write out the entries of the table that you use.
9. **Turn off all cell phones, pagers, and smartwatches**, and remove all headphones.

Problem	Points	Score
1	12	
2	12	
3	11	
4	11	
5	6	
6	8	
7	8	
8	8	
9	9	
10	15	
Total	100	

1. [12 points] Solve the following equations for the variable, showing all your work. Write your answers in **exact** form in the blank provided.

a. [4 points] $\ln(11 \cdot e^p) = -14p + 2017$

Solution:

$$\begin{aligned} -14p + 2017 &= \ln(11) + \ln(e^p) \\ &= \ln(11) + p \end{aligned}$$

$$15p = 2017 - \ln(11)$$

$p =$ $\frac{2017 - \ln(11)}{15}$

b. [4 points] $\log(10^x + 1) = \pi$.

Solution: Start by taking the base 10 exponential of both sides.

$$10^x + 1 = 10^\pi$$

$$10^x = 10^\pi - 1$$

$$x = \log(10^\pi - 1)$$

$x =$ $\log(10^\pi - 1)$

c. [4 points] $e^{t+5} = 10^t$.

Solution:

$$\ln(e^{t+5}) = \ln(10^t)$$

$$t + 5 = t\ln(10)$$

$$t(\ln 10 - 1) = 5$$

$t =$ $\frac{5}{\ln 10 - 1}$

2. [12 points] The giant cockroaches that escaped from Chuck's farm have separated into two groups. One group has settled beside Lake Popeye not far from Chickenville, the other in the caverns at Kentucky Valley. **Please leave your answers in exact form for all parts of this problem**

a. [6 points] Due to the lush vegetation and ample food near Lake Popeye, the population of cockroaches there is growing exponentially, and has increased by 11% in 6 months. How long will it take for the population to triple in size? *To receive full credit, do not assume in your computations that the initial number of cockroaches is a particular value, and don't forget to include units.*

Solution: Let $P(t) = ab^t$ be the population of cockroaches at Lake Popeye. Since the population increased by 11% in 6 months, we plug in $t = 6$ to get

$$ab^6 = 1.11a \implies b = (1.11)^{1/6}.$$

We now want to find the number of months t when the population triples. In equations, we have

$$a(1.11)^{t/6} = 3a \implies (1.11)^{t/6} = 3.$$

We solve this by taking logarithms.

$$\frac{t}{6} \ln 1.11 = \ln 3 \implies t = \frac{6 \ln 3}{\ln 1.11}.$$

The population will triple in $\frac{6 \ln 3}{\ln 1.11}$ months.

b. [3 points] The cockroaches in Kentucky Valley have not had it so easy. Their population t months after the escape can be modeled by the function

$$K(t) = 72 \cdot (6/7)^{\frac{1}{2}t-1}.$$

Find the *continuous* monthly growth rate of $K(t)$.

The continuous monthly growth rate is $\frac{1}{2} \ln \frac{6}{7}$.

c. [3 points] What is the *annual* (not continuous) growth rate of $K(t)$?

The annual growth rate is $\left(\frac{6}{7}\right)^6 - 1$.

3. [11 points] Yolko Ono purchases a serving of her favorite TV dinner, *Chuck's Caterpillar Chop and Gravy*, from Crowger's, her local supermarket chain. At home, she heats up the frozen dish in the microwave oven. Right out of the oven, the temperature of the meal is 185 °F. After 5 minutes, the meal cools to 140 °F. If left out on the counter, the meal will eventually cool to room temperature, 68 °F. **Please leave your answers in exact form for all parts of this problem.**

a. [7 points] Let $M(t) = A + Be^{kt}$ be the temperature of the meal (in degrees Fahrenheit) t minutes after it leaves the oven. Using the information given, find the values of A, B, and k.

Solution: We first solve for A and B using the value of $M(t)$ at $t = 0$ and the limiting value as t tends to infinity.

$$68 = \lim_{t\to\infty} M(t) = A$$

$$185 = M(0) = A + B = 68 + B$$

$$B = 185 - 68 = 117$$

Now we solve for k using the value of $M(t)$ at $t = 5$.

$$140 = M(5) = 68 + 117e^{k5}$$

$$e^{k5} = \frac{140-68}{117} = \frac{72}{117}$$

$$5k = \ln\frac{72}{117} \implies k = \frac{1}{5}\ln\frac{72}{117}$$

$A =$ 68

$B =$ 117

$k =$ $\frac{1}{5}\ln\frac{72}{117}$

b. [4 points] Yolko has poured a cup of hot coffee into a thick mug. The temperature of the coffee (in degrees Fahrenheit) t minutes after she pours the coffee is given by the function $C(t) = 68 + 100e^{-0.05t}$. Yolko has a sensitive beak and wants to drink the coffee when it is at 131 °F. How long does she have to wait before she can drink it?

Solution: We want to find the value of t such that $C(t) = 131$. Using the formula for $C(t)$, we get

$$131 = 68 + 100e^{-0.05t}$$

$$e^{-0.05t} = \frac{131-68}{100} = \frac{63}{100}$$

$$t = \frac{\ln\frac{63}{100}}{-0.05} = 20\ln\frac{100}{63}$$

She will have to wait $20\ln\frac{100}{63}$ minutes.

4. [11 points]

a. [5 points] Suppose that $f(y)$ is **odd** and is **periodic** of period 8 with domain $(-\infty, \infty)$. Some of its values are given in the table below.

y	0	1	2	3	4	5	6
$f(y)$	?	1.3	?	-2.9	?	?	2.2

Find the following values of f. If it is not possible to find the value specified using the information given, write NOT POSSIBLE. *You do not have to show any work for this problem.*

(i) $f(0) =$ 0

(ii) $f(-1) =$ -1.3

(iii) $f(2017) =$ 1.3

(iv) $f(2) =$ -2.2

(v) $f(4) =$ 0

b. [6 points] Suppose that $q(x) = 3e^{(x-5)^2}$ and $r(x) = e^{x^2/4}$. List the transformations you need to apply to the graph of $y = r(x)$ to transform it to that of $y = q(x)$. Fill each space with either a number or one of the phrases below, as appropriate.

SHIFT IT HORIZONTALLY TO THE RIGHT

SHIFT IT HORIZONTALLY TO THE LEFT

SHIFT IT VERTICALLY UPWARDS

SHIFT IT VERTICALLY DOWNWARDS

COMPRESS IT HORIZONTALLY

STRETCH IT HORIZONTALLY

COMPRESS IT VERTICALLY

STRETCH IT VERTICALLY

To get the graph of $y = q(x)$ starting with the graph of $y = r(x)$,

first, we SHRINK IT HORIZONTALLY by 0.5,

and then we SHIFT IT HORIZONTALLY TO THE RIGHT by 5,

and then we STRETCH IT VERTICALLY by 3.

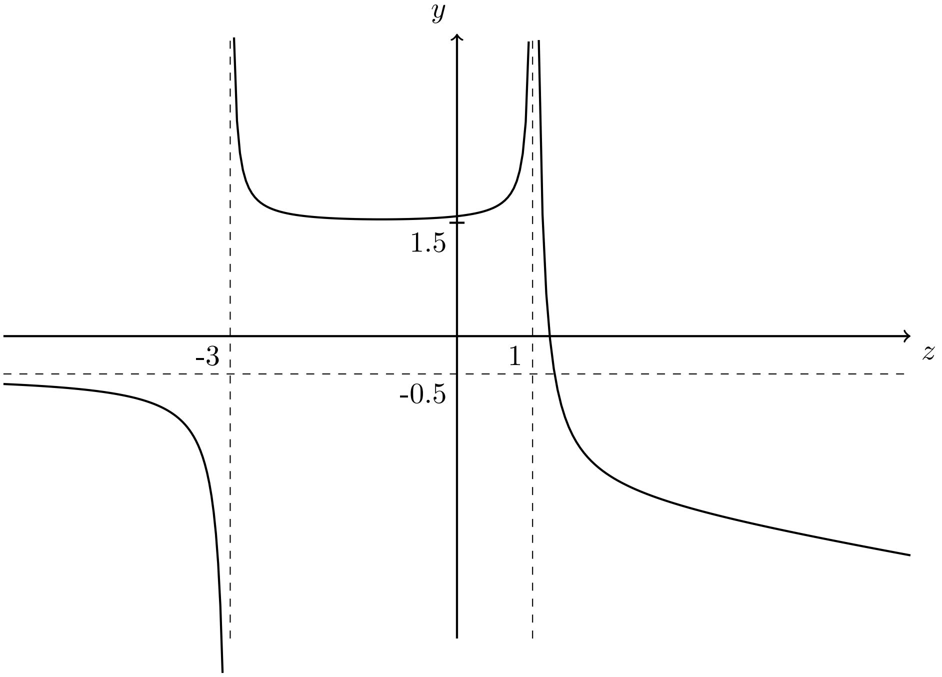
OR

first, we SHIFT IT HORIZONTALLY TO THE RIGHT by 10,

and then we SHRINK IT HORIZONTALLY by 0.5,

and then we STRETCH IT VERTICALLY by 3.

5. [6 points] Consider the following graph of a function $A(z)$. Assume the behavior of $A(z)$ depicted on the left and right of the graph continues as z approaches $-\infty$ and ∞, respectively.



a. [3 points] Write down equations for all vertical and horizontal asymptotes of $A(z)$.

The vertical asymptote(s) of $A(z)$ are $z = -3,\ z = 1$

The horizontal asymptote(s) of $A(z)$ are $w = -0.5$

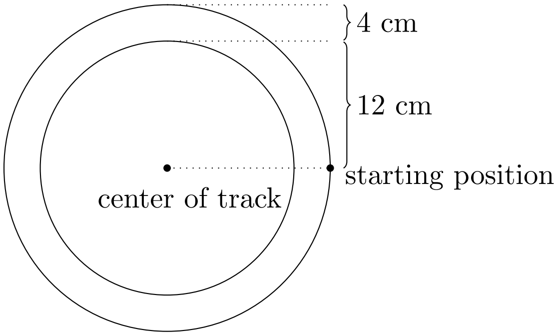
b. [3 points] Calculate the following limits.

(i) $\lim_{z \to -3^+} A(z) =$ $+\infty$

(iii) $\lim_{z \to 3^+} A(-z) =$ $-\infty$

(iv) $\lim_{z \to -\infty} 3A(z/2) =$ -1.5

6. [8 points] Gretchken has made a circular running track to test the metabolism of ants and termites receiving doses of Chemical Y. The track has an inner radius of 12 cm, and a thickness of 4 cm as depicted below. **Please leave your answers in exact form for all parts of this problem.**



a. [4 points] First, an ant runs counterclockwise following the *outer* edge of the track. If the ant runs at a constant speed of 4.8 cm/second, what is the total angular distance (in radians) that it covers in 5 minutes?

Solution: The ant travels a total distance of $4.8 \cdot 5 \cdot 60 = 1440$ cm. The radius of the circle along which it travels is $4 + 12 = 16$ cm. The total angular distance (in radians) that it covers is thus

$$\frac{1440}{r} = \frac{1440}{16} = 90.$$

The ant covers 90 radians in 5 minutes.

b. [4 points] Next, a termite run counterclockwise following the *inner* edge of the track for a total angular distance of $\frac{27\pi}{5}$ radians. How many times does it pass its starting position? What is the additional angular distance that it covers on its last, incomplete lap?

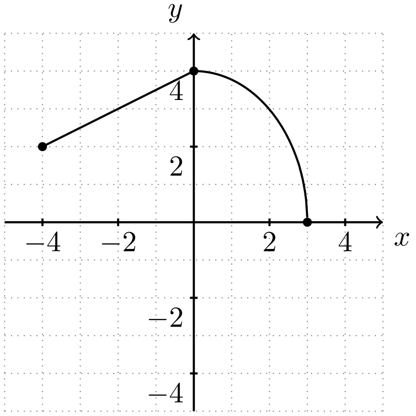
Solution:

$$\frac{27\pi}{5} = 4\pi + \frac{7\pi}{5} = 2 \cdot 2\pi + \frac{7\pi}{5}$$

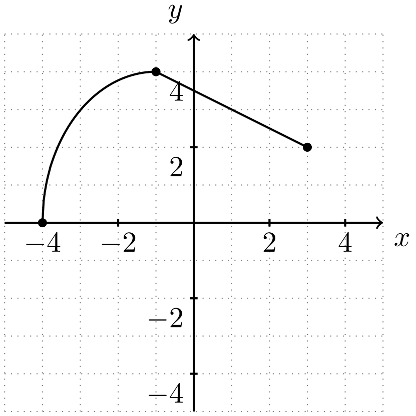
The termite passes the starting point 2 times.

It covers $\frac{7\pi}{5}$ radians after passing the starting point for the last time.

7. [8 points] Consider the following graph of a function $y = q(x)$ defined on $[-4, 3]$.

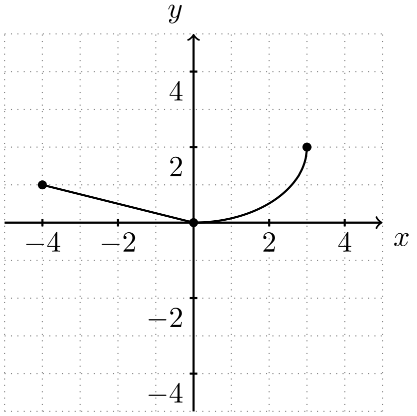


For each of the following graphs, if the graph is not a combination of shifts, stretches, compressions and reflections of the graph of $y = q(x)$, write NOT A TRANSFORMATION. Otherwise, write a formula for the function corresponding to graph in terms of $q(x)$.



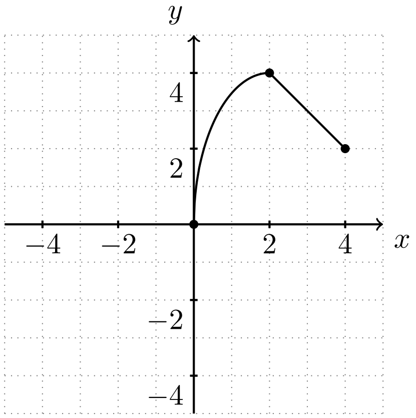
This is the graph of

$y =$ $q(-(x+1))$.



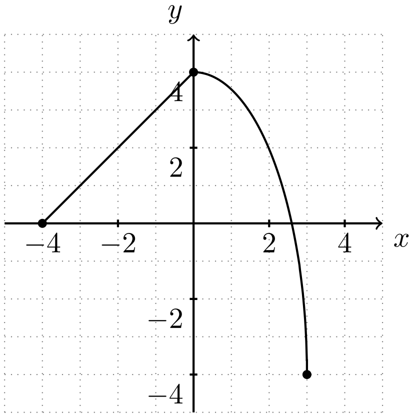
This is the graph of

$y =$ $-\frac{1}{2}q(x) + 2$.



This is the graph of

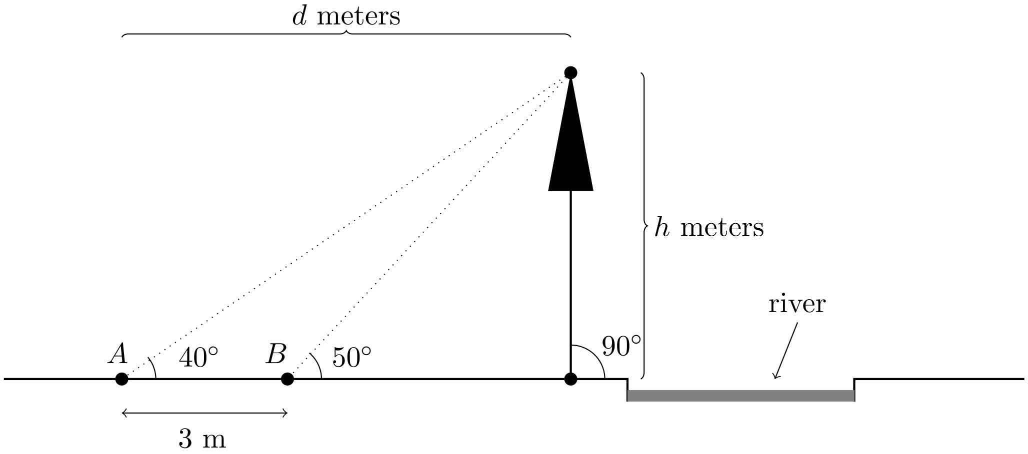
$y =$ NOT A TRANSFORMATION.



This is the graph of

$y =$ $2q(x) - 4$.

8. [8 points] Chuck had a strange yet familiar dream on the night he got back to the farm. His path was blocked by a fast-flowing river. Spying a small tree beside the river, Chuck thought about cutting down the tree to make a log bridge. He knows the points A and B are three meters apart, and he knows the angle between the ground and the line from the points A and B to the top of the tree. The tree is h meters tall and makes a right angle with the ground. The figure below depicts the situation.



Please leave your answers in exact form.

a. [4 points] Write expressions for $\tan(40°)$ and $\tan(50°)$ in terms of d and h.

$\tan(40°) =$ $\frac{h}{d}$

$\tan(50°) =$ $\frac{h}{d-3}$

b. [4 points] Solve the system of equations you got in part (a) to find h in terms of $\tan(40°)$ and $\tan(50°)$ (and not in terms of d).

Solution: We can write $h = d\tan(40°) = (d-3)\tan(50°)$. Hence

$$d(\tan(50°) - \tan(40°)) = 3\tan(50°),$$

$$d = \frac{3\tan(50°)}{\tan(50°) - \tan(40°)}.$$

Substituting d into the first equation for h, get

$$h = \frac{3\tan(40°)\tan(50°)}{\tan(50°) - \tan(40°)}.$$

$h =$ $\frac{3\tan(40°)\tan(50°)}{\tan(50°) - \tan(40°)}$

9. [9 points] The ECG of a resting cockroach can be modeled by a periodic function $v = E(t)$, which gives the electrical activity of the cockroach's heart (in millivolts) at time t (in seconds). Each cycle is comprised of a large pulse lasting 0.8 seconds immediately followed by a smaller pulse lasting 0.4 seconds. During the large pulse the electrical activity of the heart rises from 0.1 millivolts to 0.4 millivolts and then falls to 0.1 millivolts again. During the smaller pulse the electrical activity rises from 0.1 millivolts to 0.2 millivolts and then falls to 0.1 millivolts again. The cycle immediately repeats.

a. [6 points] Find the amplitude, midline, and period of $E(t)$.

Solution:

$$\frac{0.4 - 0.1}{2} = 0.15$$

$$\frac{0.4 + 0.1}{2} = 0.25$$

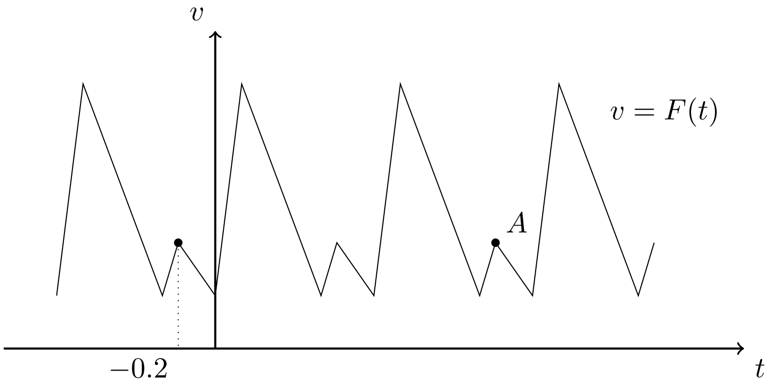
$$0.8 + 0.4 = 1.2$$

Amplitude: 0.15

Midline: $v = 0.25$

Period: 1.2

After some exercise, a cockroach's ECG can be modeled with the function $F(t) = E(\frac{4}{3}t)$. Part of the graph of $F(t)$ is shown below.



b. [3 points] Find the v- and t-coordinates of the point A.

Solution:

$$\text{Period} = 1.2 \cdot \frac{3}{4} = 0.9$$

$$-0.2 + 0.9 \cdot 2 = 1.6$$

Coordinates of A: $(1.6, 0.2)$

10. [15 points] For each of the questions below, circle **all** correct answers. You do not need to show your work for this problem. Make sure your answers are clear.

a. [3 points] The function $f(x) = \sin(x - \frac{\pi}{2})$ is

equal to $\cos(x)$ [✓ an even function] an odd function

neither even nor odd none of the above

b. [3 points] Suppose θ is an angle between 0 and 90 degrees. If $v = \sin(\theta)$, then $\cos(180° + \theta)$ is equal to

v $-v$ $\sqrt{1-v^2}$ [✓ $-\sqrt{1-v^2}$] none of the above

c. [3 points] Suppose a function $A(x)$ has a vertical asymptote of $x = 5$. The function $B(x) = 3A(3x - 6) + 1$ has a vertical asymptote of

$x = -1/3$ $x = 13/3$ $x = 15$ $x = 23/3$ [✓ none of the above]

d. [3 points] When an ant is given chemical Y, it grows to any given mass in half the time it takes for a regular ant to reach that mass. If $A(t)$ is the mass of a regular ant t weeks after it's born, and $B(t)$ is the mass of an ant given chemical Y, t weeks after it's born, which of the following equalities are true?

$A(t) = 2B(t)$ $2A(t) = B(t)$ $A(t) = B(2t)$

[✓ $A(2t) = B(t)$] none of the above

e. [3 points] Let $A > 1$ be a positive number. For which of the following intervals is the function $C(t) = A\cos(t + 1)$ concave down for the entire interval?

[✓ $[-1, 0]$] $[0, 1]$ [✓ $[\frac{3\pi}{2} - 1, \frac{5\pi}{2} - 1]$] $[\frac{3\pi}{2} + 1, \frac{5\pi}{2} + 1]$ none of the above