Math 105 — First Midterm

October 9, 2018

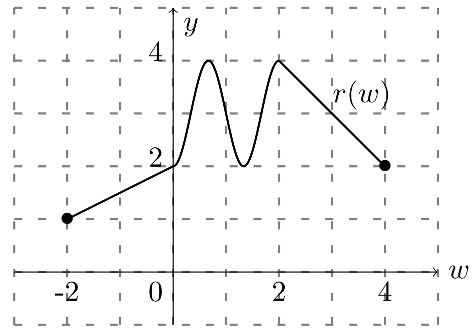
UMID: EXAM SOLUTIONS Initials: ____________

Instructor: ____________ Section: ____________

1. **Do not open this exam until you are told to do so.**
2. **Do not write your name anywhere on this exam.**
3. This exam has 10 pages including this cover. There are 10 problems. Note that the problems are not of equal difficulty, so you may want to skip over and return to a problem on which you are stuck.
4. Do not separate the pages of this exam. If they do become separated, write your UMID on every page and point this out to your instructor when you hand in the exam.
5. Please read the instructions for each individual problem carefully. One of the skills being tested on this exam is your ability to interpret mathematical questions, so instructors will not answer questions about exam problems during the exam.
6. Show an appropriate amount of work (including appropriate explanation) for each problem so that graders can see not only your answer, but also how you obtained it. Include units in your answer where that is appropriate.
7. You may use a TI-84, TI-89, TI-Nspire or other approved calculator. However, you must show work for any calculation which we have learned how to do in this course.
8. If you use graphs or tables to find an answer, be sure to include an explanation and sketch of the graph, and to write out the entries of the table that you use.
9. **Turn off all phones and smartwatches**, and remove all headphones and earbuds.

Problem	Points	Score
1	10	
2	13	
3	8	
4	4	
5	11	
6	9	
7	15	
8	8	
9	12	
10	10	
Total	100	

1. [10 points] Given below are three functions. $r(w)$ is given by a graph, $h(t)$ is given by a formula, and $n(v)$ is described verbally.



$n(v)$ has a constant rate of change, and its graph passes through the points $(1, 4)$ and $(3, 0)$.

$$h(t) = \sqrt{t-4}.$$

The function $r(w)$ is linear on $[-2, 0]$ and on $[2, 4]$. Give your answer in **exact** form (i.e. no decimal approximations) for parts **a.-c.**

a. [2 points] Complete the sentence by filling in the blank. You can express your answer in inequality or interval notation.

The domain of $h(t)$ is $\underline{\quad [4, \infty) \quad}$.

b. [2 points] Complete the sentence by filling in the blank. You can express your answer in inequality or interval notation.

The range of $r(w)$ is $\underline{\quad [1, 4] \quad}$.

c. [2 points] Complete the sentence by filling in the blank.

The average rate of change of $h(t)$ between $t = 6$ and $t = 9$ is $\underline{\dfrac{\sqrt{9-4}-\sqrt{6-4}}{9-6} = \dfrac{\sqrt{5}-\sqrt{2}}{3}}$.

d. [4 points] Find all solutions to the equation

$$n(r(w)) = -2.$$

If there is no solution, write "no solution" in the blank. Show your work. *(If needed, use the graph of $r(w)$ to give estimates for values of w in the interval $[0, 2]$. Otherwise, give your answer in exact form.)*

Solution: $n(v) = -2(v-1)+4$. Therefore, $n(r(w)) = -2(r(w)-1)+4$.

$$\begin{aligned} n(r(w)) &= -2 \\ -2(r(w)-1)+4 &= -2 \\ -2r(w)+6 &= -2 \\ -2r(w) &= -8 \\ r(w) &= 4 \end{aligned}$$

$w = \underline{\quad 2/3,\ 2 \quad}$.

2. [13 points] The table below contains some information about the functions $A(w), B(w)$ and $C(w)$.

w	-1	0	2
$A(w)$	2.5	3	4.32
$B(w)$	1.8	1	-0.6
$C(w)$	5	0	-2.5

There is **exactly one** linear and **exactly one** exponential function.

a. [7 points] In the following sentences, circle one option when it is indicated and fill in the blanks with the correct value based on the function you circled:

The function $A(w)$ / $\boxed{B(w)}$ / $C(w)$ ***(circle one)*** is linear.

The slope is ____-0.8____ and the vertical intercept is ____1____.

The function $\boxed{A(w)}$ / $B(w)$ / $C(w)$ ***(circle one)*** is exponential.

The initial value is ____3____ and the $\boxed{\text{GROWTH}}$ / DECAY ***(circle one)*** rate is ____20____%.

b. [3 points] Decide which of the functions **could** be **decreasing**. Circle **all** that apply.

$A(w)$ $\boxed{B(w)}$ $\boxed{C(w)}$ None

c. [3 points] Decide which of the functions **could** be **concave up**. Circle **all** that apply.

$\boxed{A(w)}$ $B(w)$ $\boxed{C(w)}$ None

3. [8 points] Rachel is the online marketing manager at a dress shop. She is running a week-long (168 hour) Facebook promotion for a specific dress starting Monday at 12:00am. The price of the dress changes according to how many times it has been viewed on Facebook since the start of the promotion. Let $V(h)$ be the total number of times the dress has been viewed on Facebook during the first h hours of the promotion. Let $P(v)$ be the price of the dress, in dollars, after it has been viewed v times during the promotion.

a. [3 points] Assuming P^{-1} is a function, give a practical interpretation of the expression $P^{-1}(200) = 350$.

Solution: The price of the dress is \$ 200 after it has been viewed on Facebook 350 times.

b. [3 points] Give a practical interpretation of the expression $P(V(100))$.

Solution: $P(V(100))$ is the price of the dress, in dollars, 100 hours into the promotion.

c. [2 points] Compare the quantities below by writing one of the symbols $\leq, \geq$, or $=$ in the blank, or by writing "N" if there is not enough information in the problem to compare them. You do not need to justify your answer.

$$V(20) \underline{\quad \leq \quad} V(35).$$

4. [4 points] Suppose quantities Q, E, and D are temperatures in $^\circ F$ at three different locations near Phoebe's apartment building measured at five different times during the winter. Which of Q, E and D could be a function of another?

Q	13.2	4	0	3.2	3
E	19	-1	11	17.25	-1
D	23.7	-8	15	12.3	-18

Circle all of the following statements that **could** be true:

Q is a function of E. $\boxed{Q \text{ is a function of } D.}$ $\boxed{E \text{ is a function of } D.}$

$\boxed{E \text{ is a function of } Q.}$ D is a function of E. None of these.

5. [11 points] Monica has decided to buy a restaurant that costs \$35,000. She gives \$3,600 to Chandler and asks him to find the best way to invest it, so that she will be able to buy the restaurant in at most 2 years. Chandler invests the money for Monica in a certain cryptocurrency, and the value of Monica's investment grows by exactly 10% each month. Let $G(m)$ be the value of Monica's investment (in \$) after m months.

a. [4 points] Find a formula for the function $G(m)$.

$G(m) =$ $3600(1.1)^m$

b. [3 points] Is Monica going to be able to buy the restaurant by the end of the 2 years? Circle your answer. Briefly justify your answer.

$\boxed{\text{YES}}$ NO CANNOT BE DETERMINED

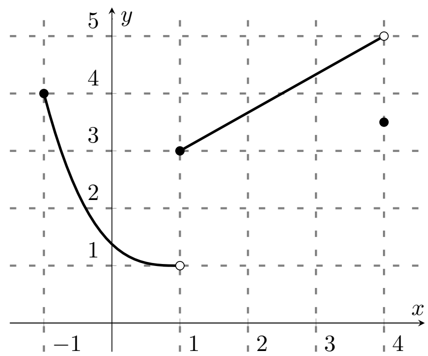
Solution: $G(24) = 3600(1.1)^{24} \approx 35459.037 > 35000$.
The value of Monica's investment will be greater than 35000 by the end of the 2 years.

c. [4 points] The owner of the restaurant decides to increase the price to \$40,000. If Monica started with the same \$3,600 initial investment, what should the minimum monthly growth rate be for Monica's investment in order for her to be able to buy the restaurant in 2 years? Show all your work, and give your answer in **exact** form.

Solution:

$$\begin{aligned} 3600(1+r)^{24} &= 40000 \\ (1+r)^{24} &= \frac{40000}{3600} \\ (1+r)^{24} &= \frac{100}{9} \\ 1+r &= \sqrt[24]{\frac{100}{9}} \\ r &= \sqrt[24]{\frac{100}{9}} - 1 \end{aligned}$$

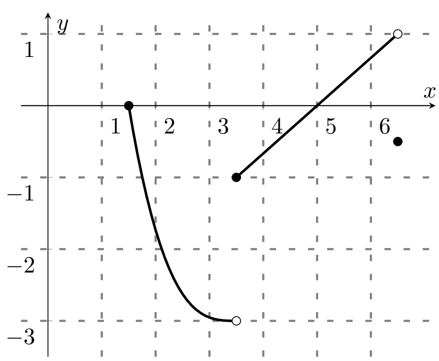
6. [9 points] The function $h(x)$ is given in the graph below. Note that h is linear for $1 \le x < 4$.



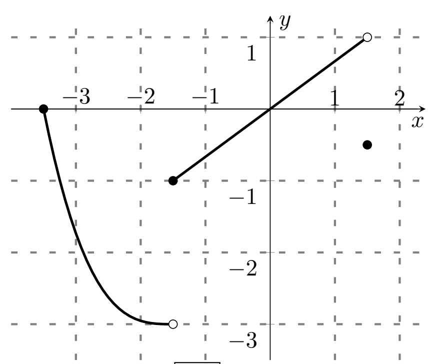
a. [3 points] Find all the values of x for which $h(x) \le 4$.

Solution: $-1 \le x \le 2.5$ or $x = 4$.

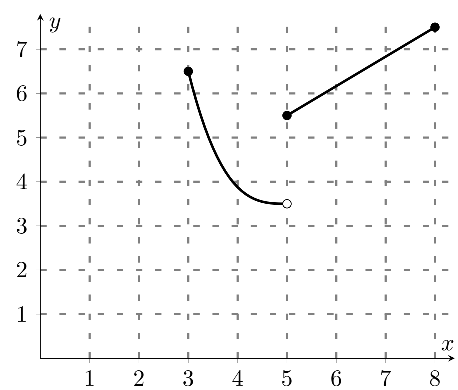
b. [2 points] Choose which of the graphs I, II, and III corresponds to the function $k(x) = h(x + 2.5) - 4$. Circle exactly one of I, II, and III.



I

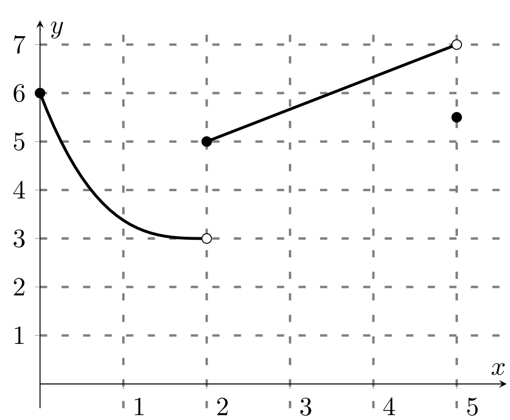


$\boxed{\text{II}}$



III

c. [4 points] Below is the graph of the function $c(x)$ which is a transformation of the graph of $h(x)$. Find a formula for $c(x)$ in terms of $h(x)$.



$c(x) =$ $h(x-1)+2$

7. [15 points] In one of his experiments, David recorded the speeds (in km/sec) of two different particles, particle A and particle B, for 8 seconds.
Let $S(t)$ be the difference between the recorded speeds of the two particles (in km/sec) t seconds after the beginning of the experiment, i.e. $S(t)$=(speed of particle A)-(speed of particle B).
David found that $S(t) = -\frac{5}{8}t^2 + 5t - 4$.

a. [5 points] Find **both coordinates** of the *maximum* of $S(t)$ by completing the square. Show your work step-by-step.

Solution:

$$\begin{aligned} S(t) &= -\frac{5}{8}t^2 + 5t - 4 \\ &= -\frac{5}{8}(t^2 - 8t) - 4 \\ &= -\frac{5}{8}(t^2 - 8t + 16 - 16) - 4 \\ &= -\frac{5}{8}(t^2 - 8t + 16) + 10 - 4 \\ &= -\frac{5}{8}(t - 4)^2 + 6 \end{aligned}$$

$S(t)$ has a *maximum* at (4,6).

b. [4 points] Find all t-values when the speeds of the two particles are equal to each other. Be sure to show your work and give you answer in **exact** form.

Solution: $S(t) = 0 \Rightarrow -\frac{5}{8}t^2 + 5t - 4 = 0$. Using the quadratic formula, we get

$$t_{1,2} = \frac{-5 \pm \sqrt{25 - 4(-4)\left(-\frac{5}{8}\right)}}{2\left(-\frac{5}{8}\right)} = \frac{-5 \pm \sqrt{15}}{-\frac{5}{4}} = 4 \pm \frac{4}{5}\sqrt{15}.$$

c. [3 points] The average rate of change of $S(t)$ between $t = 2$ and $t = 5$ is $0.625 \ \frac{\text{km/sec}}{\text{sec}}$. Give a practical interpretation for this average rate of change.

Solution: Between the second and the fifth second, the difference of the recorded speeds of the two particles increases by 0.625 km/sec every second on average.

d. [3 points] Find all t-values in the practical domain of $S(t)$ when particle B is moving faster than particle A.

Solution: We need to determine the $t-$values for which $S(t) < 0$.
t is in $[0, 4 - \frac{4}{5}\sqrt{15}) \cup (4 + \frac{4}{5}\sqrt{15}, 8]$.

8. [8 points] Find formulas for the following functions using the information given in each part. Be sure to show how you got your formula in each part. Full credit will only be given to answers with full work shown.

a. [4 points] Suppose $Q(m)$ has the following properties:

- $Q(m)$ is quadratic.
- The vertex of the graph of $Q(m-2)+5$ is (5,4).
- $Q(-1) = 2$.

Solution: The vertex of the graph of $Q(m-2)+5$ is (5,4). Therefore, the vertex of the graph of $Q(m)$ is (3,-1).

We can write: $Q(m) = a(m-3)^2 + 1$. Since $Q(-1) = 2$, we get:

$2 = a(-1-3)^2 - 1$, so $a = \dfrac{3}{16}$.

$Q(m) =$ $\dfrac{3}{16}(m-3)^2 - 1$

b. [4 points] Suppose $E(m)$ has the following properties:

- $E(m)$ is exponential.
- $\dfrac{E(-5)}{E(-3)} = \dfrac{1}{0.7225}$
- $E(-1) = 2$.

Solution: We know that $E(m) = ab^m$.

$\dfrac{1}{0.7225} = \dfrac{E(-5)}{E(-3)} = \dfrac{ab^{-5}}{ab^{-3}}$, so $b^2 = 0.7225$.

Taking a positive square root (because $b > 0$), we get $b = \sqrt{0.7225} = 0.85$.

Since $E(-1) = 2$ we get:

$a(0.85)^{-1} = 2$, so $a = 2 \cdot 0.85 = 1.7$

$E(m) =$ $1.7(0.85)^m$

9. [12 points] Joey decides to participate in a medical study that tests the effects of a new drug against fatigue on people by measuring their energy levels on a scale from 0 to 100 before and after taking the drug. The scientists conducting the study created new units of energy called energons to measure the energy level of the participants. The experiment goes as follows:

- Starting with initial energy equal to 100 energons, participants do a certain exercise for 1 hour.
- Then participants immediately take the drug and do the same exercise for one more hour.

Let $E = J(t)$ be Joey's energy level, in energons, t minutes after the beginning of the experiment.

Joey's energy level decreases at a constant rate of 0.8 energons/min during the first hour, and it decreases at a constant rate of 0.3 energons/min during the second hour.

a. [2 points] Evaluate $J(62)$.

Solution: $J(62) = 100 - 0.8 \cdot 60 - 2 \cdot 30 = 100 - 48 - 0.6 = 51.4$ energons.

b. [5 points] Write a piecewise-defined formula for the function $J(t)$ for $0 \leq t \leq 120$.

Solution:

$$J(t) = \begin{cases} 100 - 0.8t & , 0 \leq t < 60 \\ 52 - 0.3(t - 60) & , 60 \leq t \leq 120 \end{cases}$$

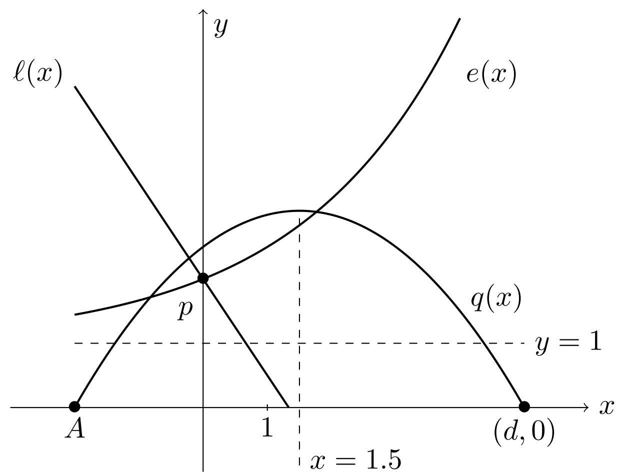
c. [5 points] Find a piecewise-defined formula for the function $t = J^{-1}(E)$.

Solution:

$$J^{-1}(E) = \begin{cases} \dfrac{70 - E}{0.3} & , 34 \leq E \leq 52 \\[2ex] \dfrac{100 - E}{0.8} & , 52 < E \leq 100 \end{cases}$$

10. [10 points] The graph below shows the functions $\ell(x), q(x)$ and $e(x)$. The letters k, p, d are unknown constants. You do not need to show your work for this problem.

- $\ell(x)$ is a linear function with formula $\ell(x) = -kx + p$.
- $\ell(x)$ has an x-intercept between 1 and 1.5.
- $e(x)$ is a transformation of an exponential function with growth **factor** k.
- $e(x)$ has a horizontal asymptote $y = 1$.
- $q(x)$ is a quadratic function with a zero at $(d, 0)$.
- The axis of symmetry of $q(x)$ is at $x = 1.5$.



a. [6 points] Circle **one** correct answer.

i.[2 points] A possible formula for $e(x)$ is:

$pk^x + 1$ $\quad$ pk^x $\quad$ $p(1+k)^x + 1$ $\quad$ $\boxed{(p-1)k^x + 1}$ $\quad$ None of these

ii.[2 points] The x-intercept of the function $\ell(x)$ is:

$\boxed{\frac{p}{k}}$ $\quad$ $\frac{k}{p}$ $\quad$ p $\quad$ 1 $\quad$ None of these

iii.[2 points] The point A is the other zero of $q(x)$. The coordinates of point A are:

$(-d, 0)$ $\quad$ $(0, -d)$ $\quad$ $(d - 1.5, 0)$ $\quad$ $\boxed{(3 - d, 0)}$ $\quad$ None of these

b. [4 points] If $q(x) = ax^2 + bx + c$ for some constants a, b and c, rank the quantities $p, 0, k, 1, a$ in ascending order:

$\underline{\quad a \quad} < \underline{\quad 0 \quad} < \underline{\quad 1 \quad} < \underline{\quad k \quad} < \underline{\quad p \quad}$