Math 105 — Final Exam

December 14, 2018

UMID: EXAM SOLUTIONS Initials: ____________

Instructor: ______________________ Section: ____________

1. **Do not open this exam until you are told to do so.**
2. **Do not write your name anywhere on this exam.**
3. This exam has 11 pages including this cover. There are 12 problems. Note that the problems are not of equal difficulty, so you may want to skip over and return to a problem on which you are stuck.
4. Do not separate the pages of this exam. If they do become separated, write your UMID on every page and point this out to your instructor when you hand in the exam.
5. Please read the instructions for each individual problem carefully. One of the skills being tested on this exam is your ability to interpret mathematical questions, so instructors will not answer questions about exam problems during the exam.
6. Show an appropriate amount of work (including appropriate explanation) for each problem so that graders can see not only your answer, but also how you obtained it. Include units in your answer where that is appropriate.
7. You may use a TI-84, TI-89, TI-Nspire or other approved calculator. However, you must show work for any calculation which we have learned how to do in this course.
8. If you use graphs or tables to find an answer, be sure to include an explanation and sketch of the graph, and to write out the entries of the table that you use.
9. **Turn off all phones and smartwatches**, and remove all headphones and earbuds.

Problem	Points	Score
1	14	
2	8	
3	10	
4	7	
5	7	
6	5	
7	10	
8	5	
9	5	
10	9	
11	10	
12	10	
Total	100	

1. [14 points] The following table contains data for the functions A, B and C. Assume that A is invertible and B is periodic with period 5.

x	-3	-2	0	1	2	3
$A(x)$	4	-7	-1	2.3	0	-0.5
$B(x)$	6.1	5.4	-1	-7	6.1	5.4
$C(x)$	0	-1	4	3	0.5	0

For parts (a)-(c) you do not *need* to show any work, but you can receive partial credit for work shown if your final answer is incorrect.

a. [2 points] Circle **all** functions that **could** be *decreasing* on the interval [1,3]:

$\boxed{A(x)}$ $B(x)$ $\boxed{C(x)}$ NONE OF THEM

b. [5 points] Evaluate the following expressions. If there is not enough information to evaluate an expression, write 'NEI':

i. [1 point] $A^{-1}(0)=$ 2

ii. [2 points] $B(-2)+B(7)=$ 11.5

iii. [2 points] $B(C(2)+0.5)=$ -7

c. [3 points] Let $D(x)=\dfrac{A(x)}{C(x)}$. Circle **all** values of x from the table that are **not** in the domain of D.

$\boxed{-3}$ -2 0 1 2 $\boxed{3}$

d. [4 points] Find **all** the x values from the table that satisfy the following equation. **Show all your work**. If there is no solution, write "NO SOLUTION".

$$B(A(x)-1)=5.4$$

Solution:

$$A(x)-1=-2 \qquad \text{or} \qquad A(x)-1=3$$
$$A(x)=-1 \qquad\qquad A(x)=4$$
$$x=0 \qquad\qquad x=-3$$

$x=$ 0, -3

2. [8 points] The parts of this problem are **unrelated** to each other. For each part find your answers in **exact** form.

a. [3 points] Let $Q = \frac{7}{4}e^{0.1(t-2)}$. Find the initial value and the growth factor of Q.

Initial value: $\frac{7}{4}e^{-0.2}$

Growth factor: $e^{0.1}$

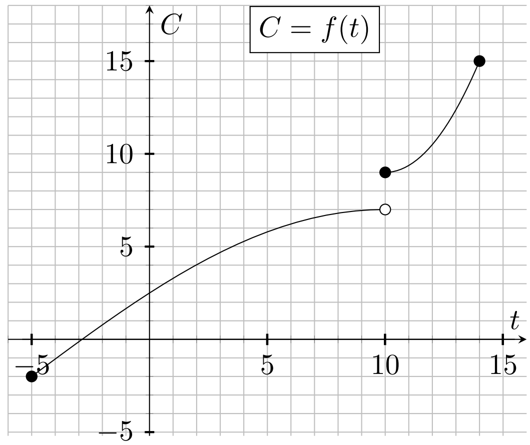
b. [5 points] Let $p(x)$ be a power function that passes through the points (5,8) and (10,32). Find a formula for $p(x)$. Be sure to **show all your work**.

Solution: The function $p(x)$ has the form kx^p for some constants k, p. Using the two points given we get:

$8 = k \cdot 5^p$ and $32 = k \cdot 10^p$.
Therefore, $\frac{32}{8} = \frac{10^p}{5^p}$ or equivalently $4 = 2^p$ which leads to $p = 2$.
Now by substituting $p = 2$ to the first equation, we obtain $8 = k \cdot 25$ and finally get $k = \frac{8}{25}$.

$p(x) =$ $\frac{8}{25}x^2$

3. [10 points] Let $C = f(t)$ be a piecewise-defined and invertible function for $-5 \le t \le 14$. Below is given the graph of f.
Note that f is concave down on [-5,10) and concave up on [10,14].



a. [5 points] Fill in the blanks:

i. [2 points] Give the range of f using **interval notation**: $[-2, 7) \cup [9, 15]$.

Note that part ii is about f^{-1}, **NOT** f. *You may estimate your answer if needed.*

ii. [3 points] The average rate of change of f^{-1} on [4,6] is $\approx$ $\frac{3.5}{2}$.

b. [5 points] Let $g(t) = -f(0.4t + 5)$.

i. [3 points] Find the domain of g. Give your answer using interval notation:

Domain of g: $[-25, 22.5]$

ii. [2 points] Circle **only one** of the four options listed below to complete the following sentence:

On the interval [-8,-5] the function g is ...

increasing and *concave up*. | *decreasing and concave up.*

decreasing and *concave down*. | *increasing* and *concave down*.

4. [7 points] **In this problem you do not *need* to show any work, but you can receive partial credit for work shown if your final answer is incorrect.** Write your final answer in the space provided.

Consider the function:

$$P(r) = \frac{r(2r-1)^3}{5(2r-1)(r^2+10)(r+3)}$$

a. [3 points] Find the $r-$**coordinate(s)** of the hole(s) and the zero(s) of $y = P(r)$. If the function has no holes or zeros, write NONE in the space provided.

$r-$coordinate(s) of hole(s): 0.5

$r-$coordinate(s) of zero(s): 0

b. [4 points] Find the **equations** of the vertical and horizontal asymptote(s) of $y = P(r)$. If the function has no vertical or horizontal asymptotes, write NONE in the space provided.

Vertical asymptote(s): $r = -3$

Horizontal asymptote(s): $y = \frac{8}{10}$

5. [7 points] Ross is playing "Dinomite 2" again. In round 2018 he is given that the population of the Gigantosaurus t years after 65 million years ago can be modeled by the following function:

$$G(t) = 47 + 38\cos(\pi(t-3))$$

Help Ross find all values of t on the interval [3,6.5] for which the population of the Gigantosaurus is equal to 77. You should show **all your work** for this problem and give your answer in **exact** form.

Solution:

$$\begin{aligned} 77 &= 47 + 38\cos(\pi(t-3)) \\ \frac{30}{38} &= \cos(\pi(t-3)) \\ \pi(t-3) &= \arccos\left(\frac{30}{38}\right) \\ t &= \frac{1}{\pi}\arccos\left(\frac{30}{38}\right) + 3 \end{aligned}$$

By using the symmetry of the graph we get that the solutions that lie on the interval $[3, 6.5]$ are:

$t=$ $\frac{1}{\pi}\arccos\left(\frac{30}{38}\right) + 3,\ \frac{1}{\pi}\arccos\left(\frac{30}{38}\right) + 5,\ 5 - \frac{1}{\pi}\arccos\left(\frac{30}{38}\right)$

6. [5 points] Joey is taking a road trip from New York to Los Angeles to continue his acting career. The computer in his car calculates that when the car's speed is v miles per hour (mph), the car uses

$$g = f(v) = \frac{1}{20}\log(27 \cdot 10^v)$$

gallons of gas per hour. Assume the domain of $f(v)$ is $(0, 150]$.
Find a formula for $f^{-1}(g)$.

Solution:

$$\begin{aligned} g &= \frac{1}{20}\log(27 \cdot 10^v) \\ 20g &= \log(27 \cdot 10^v) \\ 20g &= \log(27) + \log(10^v) \\ 20g - \log(27) &= \log(10) \cdot v \\ v &= 20g - \log(27) \end{aligned}$$

$f^{-1}(g)=$ $20g - \log(27)$

7. [10 points] Rachel is Phoebe's secret snowflake this year, so she decided to make a guitar for her. Rachel needs to buy some wood and she has narrowed down her options to three: mahogany, walnut and cedar.
Let $M(v)$, $W(v)$ and $C(v)$ be the costs, in dollars, for buying v kg of the mahogany, walnut and cedar wood, respectively.
In addition, assume that the functions M, W and C are invertible.

a. [3 points] Write a mathematical expression for the following:

The total cost, in **cents**, of buying 3 kg of cedar and 2.5 kg of walnut.
(*Note: There are 100 cents in 1 dollar.*)

Solution: $100(C(3) + W(2.5))$

b. [7 points] Write a practical interpretation of the following:

i. [3 points] $C^{-1}(20)$

Solution: $C^{-1}(20)$ is the amount of of cedar, in kg, Rachel can buy with \$20.

ii. [4 points] $W(6) = M(4)$.

Solution: The cost, in dollars, of buying 6 kg of walnut and 4 kg of mahogany is the same.

8. [5 points] Consider the polynomial: $G(x) = x^5 - 6x^3 + 9x$. Find the zero(s) of G. Your answer should be **exact**, and must be found *algebraically*. If there are no zeros, write NONE in the space provided:

Solution:

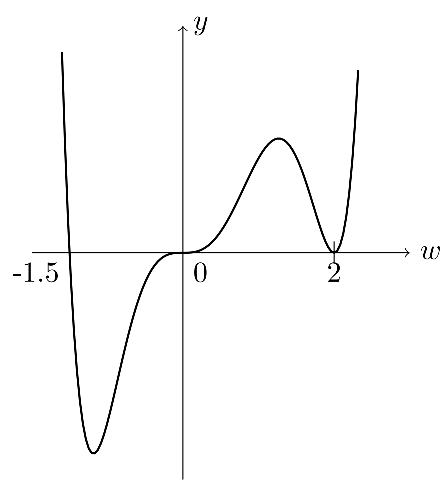
$$x^5 - 6x^3 + 9x = 0$$
$$x(x^4 - 6x^2 + 9) = 0$$

Then, $x = 0$ or

$$x^4 - 6x^2 + 9 = 0$$
$$(x^2 - 3)^2 = 0$$
$$x^2 - 3 = 0$$
$$x^2 = 3$$
$$x = \sqrt{3} \quad \text{or} \quad x = -\sqrt{3}$$

Zero(s): $0, \sqrt{3}, -\sqrt{3}$

9. [5 points] Below is part of the graph of a polynomial $P(w)$. Assume that the point (1,1.25) lies on the graph of $y = P(w)$ and $P(w)$ has **exactly** three distinct zeros. Find a possible formula for $P(w)$ so that it has the **smallest** degree possible. Show your work carefully.



Solution: We see that there are 3 distinct zeros, namely -1.5, 0 and 2, with multiplicities 1, an odd number (greater than one) and an even number respectively.

In order for the polynomial to have the smallest degree, we need to pick the smallest multiplicities possible for each zero, i.e. 1,3 and 2.

So, $P(w) = Aw^3(w + 1.5)(w - 2)^2$ for some constant A.

Since $P(1) = 1.25$ we get that $1.25 = 2.5A$ which implies that $A = \frac{1}{2}$.

$P(w) =$ $0.5w^3(w + 1.5)(w - 2)^2$

10. [9 points] For each part of this problem, circle **all** of the expressions which could be formulas for the function described. There could be more than one answer for each part.

a. [3 points] The function $f(x)$ satisfies $\lim_{x\to-\infty} f(x) = +\infty$. Then $f(x)$ **could** be:

$7x$ $\tan(5\pi x)$ $\boxed{3^{-x}}$ $\boxed{x^4+6}$

$\ln(10x-1)$ $\sin\left(\frac{\pi}{2}(x+1)\right)$ NONE OF THESE

b. [3 points] The function $k(x)$ has a vertical asymptote at $x = \frac{1}{10}$. Then $k(x)$ **could** be:

$7x$ $\boxed{\tan(5\pi x)}$ 3^{-x} x^4+6

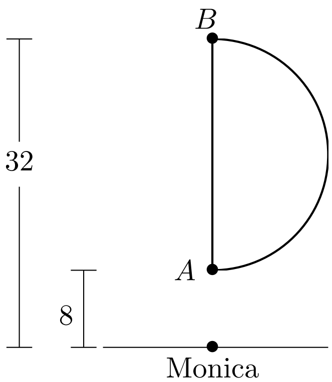
$\boxed{\ln(10x-1)}$ $\sin\left(\frac{\pi}{2}(x+1)\right)$ NONE OF THESE

c. [3 points] The function $j(x)$ is periodic with period 4. Then $j(x)$ **could** be:

$7x$ $\tan(5\pi x)$ 3^{-x} x^4+6

$\ln(10x-1)$ $\boxed{\sin\left(\frac{\pi}{2}(x+1)\right)}$ NONE OF THESE

11. [10 points] Chandler wants to lose some weight after Thanksgiving and he asks Monica to coach him. His task for today is to jog **once around** a semicircular path shown in the picture below.



Chandler starts running along the arc from point A to point B and then along the straight path back to point A. He runs at a constant speed of $\frac{2\pi}{3}$ meters per second the whole time. Monica is standing 8 meters away from point A and 32 meters away from point B.

Suppose t represents the number of seconds after Chandler began to jog.

a. [3 points] For what values of t is Chandler running along the **arc** AB? You can use interval notation or inequalities.

Solution: The length of the arc AB is $s = 12\pi$ meters. If we denote Chandler's (constant) speed as v, then the time needed to cover the arc is: $T = \frac{s}{v} = \frac{12\pi}{\frac{2\pi}{3}} = 18$ seconds.

b. [4 points] While Chandler runs along the **arc** AB, $d(t)$ is the **vertical** distance between his location and the line Monica is standing on t seconds after he started jogging. Find a formula for $d(t)$. (Note that the domain of $d(t)$ should be the t values you found in part (a).)

Solution:

The function $d(t)$ is sinusoidal. The minimum and maximum values are 8 and 32 respectively and the period is 36 (according to the answer from part a). A possible formula for $d(t)$ is $20 - 12\cos(\frac{\pi}{18}t)$.

$d(t) =$ $20 - 12\cos(\frac{\pi}{18}t)$, for $0 \leq t \leq 18$.

c. [3 points] While Chandler runs along the **straight path** BA, $\ell(t)$ is the **vertical** distance between Chandler and the line Monica is standing on t seconds after he started jogging. Find a formula for $\ell(t)$.

Solution: The function $\ell(t)$ has to be a linear function, since Chandler is running along a straight path with constant speed. The slope of the linear function will be $-\frac{2\pi}{3}$ since Chandler is moving from B to A. Now using the fact that Chandler's distance from the line Monica is standing on is 32 meters at $t = 18$, we can use point-slope formula and conclude that: $\ell(t) = -\frac{2\pi}{3}(t - 18) + 32$. The time Chandler needs to go from B back to A is $\frac{24}{\frac{2\pi}{3}} = \frac{36}{\pi}$, where 24 is the length of the line segment BA.

$\ell(t) =$ $-\frac{2\pi}{3}(t - 18) + 32$, for $18 \leq t \leq 18 + \frac{36}{\pi}$.

12. [10 points] In the following sentences circle **all** that apply. There might be more than one correct choice for each part.

a. [3 points] The function $y = r(x)$ has a horizontal asymptote at $y = 0$. The formula of $r(x)$ could be:

$\boxed{\frac{x^{2018}}{e^{0.01x}}}$ $\boxed{x^{-\frac{1}{2}}}$ NONE OF THESE

$\frac{x(x-2)(x+1)}{5x^3}$ $(0.8)^x - 2$

b. [3 points] The equation $\tan(\frac{x}{2} + \pi) = 5$ has solution:

$\arctan(5) - \pi$ $\boxed{2\arctan(5) - 2\pi}$ NONE OF THESE

$\boxed{2\arctan(5)}$ $2\arctan(5) + \pi$

c. [2 points] Let $Q(x)$ be an **odd** function such that $\lim_{x\to 5^-} Q(x) = -\infty$. Then $\lim_{x\to -5^+} Q(x)$ is equal to:

$-\infty$ 0 $\boxed{+\infty}$ 5 NONE OF THESE

d. [2 points] Let $g(x)$ be a function that has domain $[0, \infty)$ and $f(x) = x^3 + x^2$. The domain of $g(f(x))$ is:

$[0, \infty)$ $\boxed{[-1, \infty)}$ all real numbers $(-\infty, 1]$ NONE OF THESE