Math 105 — Final Exam

December 17, 2019

UMID: ____________________ Initials: __________

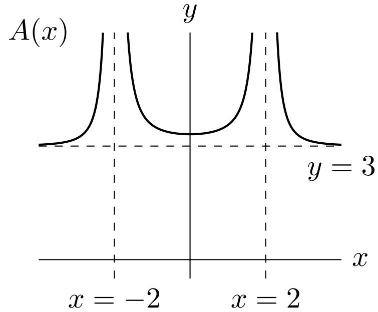
Instructor: ____________________ Section: __________

1. **Do not open this exam until you are told to do so.**
2. **Do not write your name anywhere on this exam.**
3. This exam has 10 pages including this cover. There are 11 problems. Note that the problems are not of equal difficulty, so you may want to skip over and return to a problem on which you are stuck.
4. Do not separate the pages of this exam. If they do become separated, write your UMID on every page and point this out to your instructor when you hand in the exam.
5. Please read the instructions for each individual problem carefully. One of the skills being tested on this exam is your ability to interpret mathematical questions, so instructors will not answer questions about exam problems during the exam.
6. Show an appropriate amount of work (including appropriate explanation) for each problem so that graders can see not only your answer, but also how you obtained it. Include units in your answer where that is appropriate.
7. You may use a TI-84, TI-89, TI-Nspire or other approved calculator. However, you must show work for any calculation which we have learned how to do in this course.
8. If you use graphs or tables to find an answer, be sure to include an explanation and sketch of the graph, and to write out the entries of the table that you use.
9. Any work that is erased or crossed out will not be graded.
10. **Turn off all cell phones, pagers, and smartwatches**, and remove all headphones.

Problem	Points	Score
1	9	
2	11	
3	10	
4	17	
5	4	
6	6	
7	4	
8	5	
9	12	
10	10	
11	12	
Total	100	

1. [9 points]

a. [4 points] Let $A(x)$ be the function graphed below with end behavior shown. $A(x)$ has vertical asymptotes at $x = -2$ and $x = 2$, and it has a horizontal asymptote at $y = 3$.



Find the domain and range of $\ln(A(x))$. Give your answer in interval notation, using exact form for any numbers in the endpoints of your interval(s).

The domain of $\ln(A(x))$ is ________________________.

The range of $\ln(A(x))$ is ________________________.

b. [5 points] The table below has some values of the function $B(t)$.

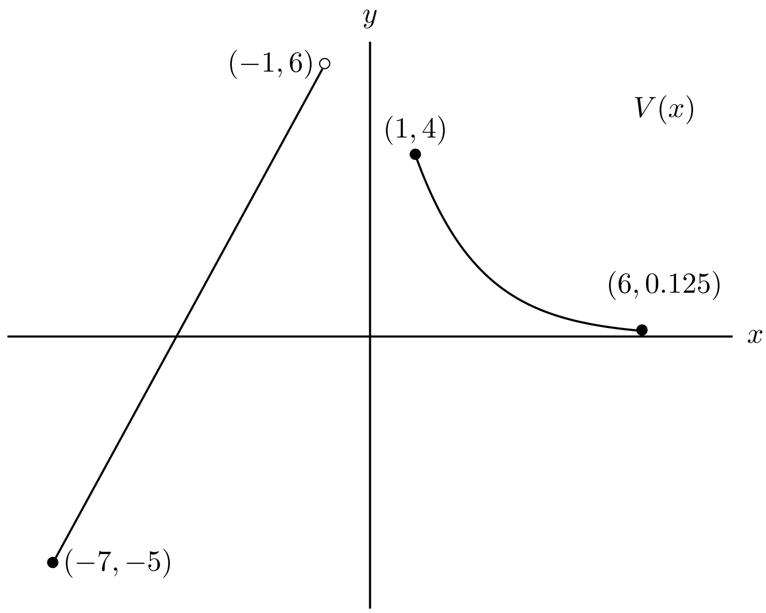
t	0	1	3	7	8	9	11
$B(t)$	3	2	6	4	12	3	1

Find all solutions of the equation

$$(B(t))^2 - 7B(t) + 12 = 0$$

that can be determined using only the information in the table above. Circle your final answer(s).

2. [11 points] Let $V(x)$ be a function whose graph is pictured below. It has two pieces - one piece is a **linear** function and one is an **exponential** function. Do not assume anything about this function outside of the part shown in the graph below.

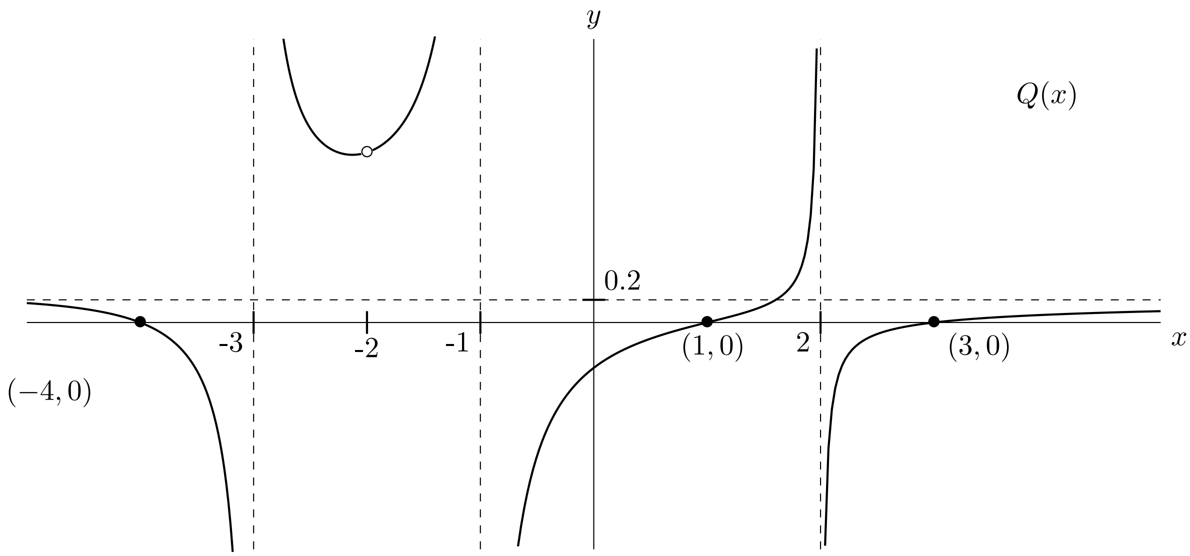


Write a piecewise-defined formula for $V(x)$. For this problem you will be graded both on the correctness of your formulas for each piece and on the use of piecewise notation. Circle your final answer for $V(x)$.

3. [10 points] Below is the graph of a rational function $Q(x)$. Note that

- $Q(x)$ has a horizontal asymptote at $y = 0.2$
- $Q(x)$ has zeros at $x = -4$, $x = 1$, and $x = 3$
- $Q(x)$ has a hole at $x = -2$
- $Q(x)$ has vertical asymptotes at $x = -3$, $x = -1$, and $x = 2$

Using the information in the portion of the graph shown, write a possible formula for $Q(x)$. You do not need to simplify your answer.



$Q(x) =$ ______________________________

4. [17 points] The following table contains information about the functions $F(x)$, $G(x)$, and $H(x)$. The functions satisfies the following properties:

- $F(x)$ is a power function.
- $G(x)$ is an odd, periodic function with period 7.
- $H(x)$ is a quadratic function with average rate of change -1.5 on $[0, 5]$

Assume that all three functions are defined for all real numbers.

x	2	5	8
$F(x)$	$\frac{3}{5}$	$\frac{75}{8}$	$\frac{192}{5}$
$G(x)$	4	?	11
$H(x)$	0.4	-3.5	-5.6

a. [8 points] Compute the following values. If it cannot be determined, write NEI. You don't need to show work on this part of the problem, but you could receive partial credit for work shown.

- $F(0) =$ ________________
- $G(0) =$ ________________
- $G(1) =$ ________________
- $G(5) =$ ________________
- $G(-8) =$ ________________
- $H(0) =$ ________________

b. [6 points] Find a formula for $F(x)$. Circle your answer.

c. [3 points] What is the sign of the leading coefficient of $H(x)$? Give a brief justification of how you determined it.

5. [4 points]
Let α and β be constants such that

- $\ln(\alpha) = 2$
- $\ln(\beta) = 5$

Find the value of $\ln(\alpha^6\beta^{-3}e^{25})$. Your answer should **not** include α, β, or ln.

$\ln(\alpha^6\beta^{-3}e^{25}) =$ ____________________

6. [6 points] Let $P(x)$ be a polynomial with the following properties:

- $P(x)$ only has zeros at $x = -3, -1, 2$
- $P(x)$ has degree 4
- The graph of $P(x)$ passes through the points $(-4, -36)$ and $(-2, -8)$

Find a formula for $P(x)$. You do not need to simplify your answer.

$P(x) =$ ____

7. [4 points] Find all possible solutions w to the equation $\dfrac{2e^{9(w-1)}}{7} = 3$. Be sure to show all your steps, give your answer in **exact** form, and **circle** your final answer.

8. [5 points] Consider the function

$$R(q) = \frac{5q+3}{4-q}$$

Find a formula for its inverse. Be sure to show all your steps, and **circle** your final answer.

9. [12 points]

a. [6 points] While searching for cryptids, Roy claims he found a secret island with crazy thermodynamic properties. According to him, the temperature on the island fluctuates in a 24 hour cycle that can be modeled by a sinusoidal function. The maximum temperature of 45° Celsius occurs at 1 p.m. every day, and the minimum temperature of −25° Celsius occurs at 1 a.m. every day. Let the sinusoidal function $C(t)$ be the temperature, in degrees Celsius, on the island t hours after 8 a.m. Find a formula for $C(t)$.

b. [6 points] On the island, Roy also claims to have found a population of the elusive Megaconda! In his notes, he writes that it is clear that the population size of Megaconda population must fluctuate in a sinusoidal manner, and that there are $M(t)$ thousand Megacondas t months after his discovery. Let

$$M(t) = 13 \sin\left(\frac{\pi t}{3}\right) + 25$$

Find the first two times after Roy's discovery when the Megaconda population is 18,000. Give your answers using **exact** form.

10. [10 points] When not selling cards, Rowena runs a rather popular ice cream shop in town. Her store carries only two flavors, mango and strawberry, which she sells for $M(k)$ and $S(k)$ dollars, respectively, for k kilograms. Assume that both functions are invertible, but **do not** assume anything else about them. Your answers for this problem may involve M, S, or their inverses.

a. [2 points] Give a practical interpretation of $S^{-1}(4.7)$.

b. [3 points] Give a practical interpretation of $M^{-1}(S(1.5)) = 1$.

c. [2 points] Write an equation that expresses the following: "7 kg of strawberry ice cream costs 4 dollars less than 5 kg of mango ice cream."

d. [3 points] A customer bought T total kg of ice cream at Rowena's shop. If they spent \$20 on strawberry ice cream, find an expression for the amount, in dollars, they spent on mango ice cream. Your answer may involve T.

11. [12 points] For each of the questions below, circle **all** solutions that are correct.

a. [3 points] Let $Q(x) = \dfrac{(3+2x)(6x^2-9)}{(3x^2+1)(7-x)}$.

What are the **horizontal** asymptote(s) of $2Q(3x+6)+7$?

$y=-1$ $\quad$ $y=3$ $\quad$ $y=-6$ $\quad$ $y=-11$

$y=-4$ $\quad$ $y=\dfrac{12}{7}$ $\quad$ None of these

b. [3 points] If $\sin(x) = \frac{4}{5}$, then what value(s) can $\cos(x)$ be?

$\dfrac{3}{5}$ $\quad$ $\dfrac{1}{3}$ $\quad$ $-\dfrac{3}{5}$ $\quad$ $\dfrac{\sqrt{3}}{2}$

$-\dfrac{\sqrt{3}}{2}$ $\quad$ $-\dfrac{1}{3}$ $\quad$ None of these

c. [3 points] The function $f(x)$ has the property $\lim_{x\to\infty} f(x) = \infty$. Which of the following could be $f(x)$?

$\ln(x)$ $\quad$ $\dfrac{.001e^x}{30x^{100}+14x^{200}}$ $\quad$ $e^{\sin(x)+\cos(x)}$ $\quad$ $\dfrac{x^{\frac{1}{2}}+4}{(\ln(x))^4 - x^{\frac{2}{3}}}$

x^{-2} $\quad$ $\dfrac{x^4+3x^2+7}{3x^3+x+x^5}$ $\quad$ None of these

d. [3 points] Which functions are periodic with period 4?

$5\sin(\frac{\pi}{2}(x-3))+1$ $\quad$ $4\cos(\frac{2}{\pi}(x+2))$ $\quad$ $\tan(\frac{\pi x}{4})$ $\quad$ $e^{\cos(4x)}$

$\tan(\frac{\pi x}{2})+4$ $\quad$ $e^{\sin(\frac{2x}{\pi})}$ $\quad$ None of these