Math 105 — First Midterm

October 8, 2019

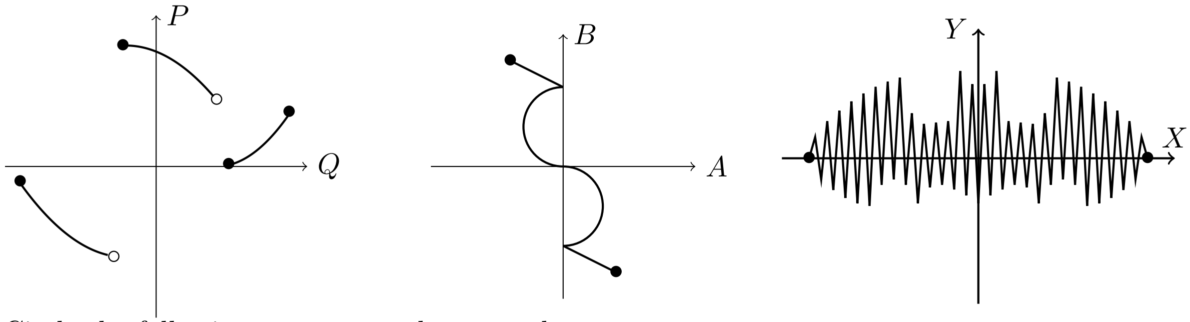
UMID: EXAM SOLUTIONS Initials: ____________

Instructor: ____________ Section: ____________

1. **Do not open this exam until you are told to do so.**
2. **Do not write your name anywhere on this exam.**
3. This exam has 9 pages including this cover. There are 10 problems. Note that the problems are not of equal difficulty, so you may want to skip over and return to a problem on which you are stuck.
4. Do not separate the pages of this exam. If they do become separated, write your UMID on every page and point this out to your instructor when you hand in the exam.
5. Please read the instructions for each individual problem carefully. One of the skills being tested on this exam is your ability to interpret mathematical questions, so instructors will not answer questions about exam problems during the exam.
6. Show an appropriate amount of work (including appropriate explanation) for each problem so that graders can see not only your answer, but also how you obtained it. Include units in your answer where that is appropriate.
7. You may use a TI-84, TI-89, TI-Nspire or other approved calculator. However, you must show work for any calculation which we have learned how to do in this course.
8. If you use graphs or tables to find an answer, be sure to include an explanation and sketch of the graph, and to write out the entries of the table that you use.
9. **Turn off all cell phones, pagers, and smartwatches**, and remove all headphones.

Problem	Points	Score
1	5	
2	8	
3	10	
4	8	
5	10	
6	9	
7	15	
8	12	
9	15	
10	8	
Total	100	

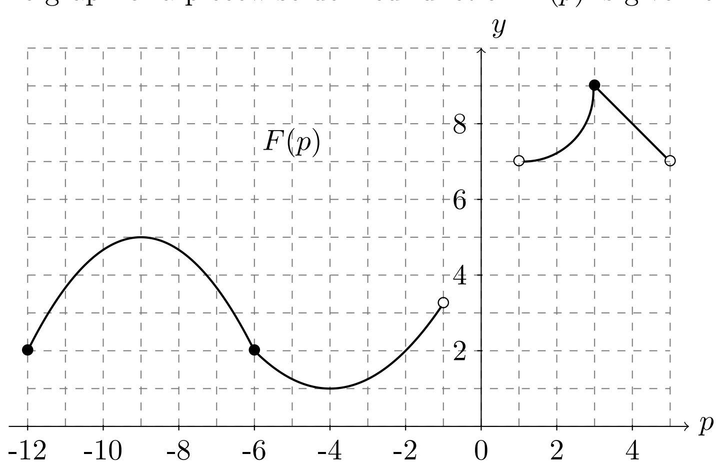
1. [5 points] Consider the three graphs given below that give relationships between the variables on their axes. Assume all information about each relationship is shown in the graphs.



Circle the following statements that must be true:

$\boxed{P \text{ is a function of } Q.}$	B is a function of A.	$\boxed{Y \text{ is a function of } X.}$
$\boxed{Q \text{ is a function of } P.}$	$\boxed{A \text{ is a function of } B.}$	X is a function of Y.

2. [8 points] The entire graph of a piecewise-defined function $F(p)$ is given below:



For both parts below, if necessary, you may estimate values that cannot be determined exactly from the graph.

a. [4 points]
Using **interval notation**, write the domain and range of $F(p)$.

Domain: $[-12, -1) \cup (1, 5)$

Range: $[1, 5] \cup (7, 9]$

b. [4 points]
Using **interval notation**, find the p-values where $F(p)$ is concave up and concave down.

Concave Up: $[-6, -1) \cup (1, 3]$

Concave Down: $[-12, -6]$

3. [10 points] Suppose:

- $f(x)$ is a function with domain $(-2, 5]$.
- $g(x) = f(x+5) + 4$.
- $h(x) = 3 + 2^x$.
- $j(x) = -7 + 0.6^x$.

You do not need to show any work for this problem, but you may receive partial credit for correct work shown. Please be sure to **circle** your answers in all parts of this problem.

a. [3 points]
What is the domain of $g(x)$? Give your answer using **inequalities**.

Solution: The graph of $g(x)$ is the graph of $f(x)$ shifted 5 units to the left, so the domain of $g(x)$ can be obtained by shifting the domain of $f(x)$ accordingly, giving us

$$-7 < x \leq 0$$

b. [3 points]
The **point** $(4, -7)$ lies on the graph of $f(x)$. What point MUST lie on the graph of $g(x)$?

Solution: Since the graph of $g(x)$ is a shift of the graph of $f(x)$ by 5 units left and 4 units up, the related point on $g(x)$ is the point $(4, -7)$ shifted the same way, giving us

$$(-1, -3)$$

c. [2 points]
The horizontal asymptote of $y = h(x)$ is:

Solution: The horizontal asymptote of 2^x is $y = 0$, and since the graph of $h(x)$ is the graph of 2^x shifted up by 3, $h(x)$ has horizontal asymptote

$$y = 3$$

d. [2 points]
$\lim_{x\to\infty} (j(x)) =$

Solution: We know $\lim_{x\to\infty} 0.6^x = 0$, and since the graph of $j(x)$ is the graph of 0.6^x shifted down by -7, we get that

$$\lim_{x\to\infty} j(x) = -7$$

4. [8 points] A new cryptocurrency ExpCoin was created to have its value grow exponentially over time. The value, in dollars, of one ExpCoin t years after ExpCoin was invented is given by

$$V(t) = 900(3)^{2t-2}.$$

Fill in the blanks below with correct numbers given in **exact form**.

a. [2 points]

One ExpCoin was worth \$ 100 when ExpCoin was invented.

b. [2 points]
The **yearly growth factor** of ExpCoin is 9.

c. [4 points]
The value of one ExpCoin grows by $100(9^{1/365} - 1)$ % per **day**. Note that this problem is about the **daily** not yearly growth rate. Assume for this problem that there are 365 days in one year.

Solution: Note that the daily growth factor is $9^{1/365}$, hence the result above.

5. [10 points] At Rowena's trading card store, she sells regular cards and foil cards. All the cards are rated on their rarity R which is a number between 0 and 15. A regular card of rarity R costs $h(R)$ dollars, while a foil card of rarity R costs $f(R)$ dollars. Suppose both $h(R)$ and $f(R)$ have inverse functions.

a. [3 points] Give a practical interpretation of the expression $h^{-1}(12)$.

Solution: $h^{-1}(12)$ is the rarity of a card worth \$12.

b. [3 points]
Write an equation, possibly involving the functions h and f, that expresses the following: "A regular card of rarity 7 costs \$100 more than twice the cost of a foil card of rarity 3."

Solution: $h(7) = 100 + 2f(3)$

c. [4 points]
Give a practical interpretation of the equation $h(f^{-1}(729)) = 180$.

Solution: A regular card, with the same rarity of a foil card that costs \$729, costs \$180.
or:
A foil card worth \$729 would be worth \$180 if it was a regular card.

6. [9 points] Zunari wants to bake bread to sell in his store. However, he is not a baker, and despite knowing individual conversions, forgets how to put them all together. Here is what he knows:

- The weight of f cups of flour is $K(f)$ kilograms.
- From q kilograms of flour, he can make $D(q)$ kilograms of dough.
- x kilograms of dough weigh $P(x)$ pounds.
- From x kilograms of dough, he can make $L(x)$ loaves of bread.
- Selling ℓ loaves of bread, he collects a total of $C(\ell)$ dollars.

Formulas for each of the functions above are given in the following table. Assume the inputs for all functions in the table must be at least one.

$K(f)$	$0.14f$
$D(q)$	$3.5q - 1$
$P(x)$	$2.2x$
$L(x)$	$2x - 0.5$
$C(\ell)$	$4\ell + 2$

a. [2 points] Suppose Zunari can make $H(f)$ loaves of bread from f cups of flour, where $H(f)$ is a composition of the above functions. Express $H(f)$ as a composition of functions.

Solution:
$H(f) = L(D(K(f)))$

b. [3 points]
Find an explicit formula for $H(f)$.
(i.e. Your answer should not involve any of the letters K, D, P, L, C).

Solution:

$$\begin{aligned} H(f) &= 2(3.5(.14f) - 1) - .5 \\ &= .98f - 2.5 \end{aligned}$$

c. [4 points]
If $M = C(\ell)$, find an explicit formula for $C^{-1}(M)$.

Solution: To solve for the inverse, we solve for the output variable in terms of the input variable.

$$\begin{aligned} M &= 4\ell + 2 \\ M - 2 &= 4\ell \\ \frac{M-2}{4} &= \ell = C^{-1}(M) \end{aligned}$$

7. [15 points] In the table below, there is at least one function that could be exponential and one that could be linear.

q	1	4	5
$A(q)$	17	$\frac{11}{3}$	5
$B(q)$	$\frac{8}{3}$	9	$\frac{27}{2}$
$C(q)$	125	25	1
$D(q)$	$\frac{3}{2}$	2	$\frac{13}{6}$

a. [3 points]
Which of the above functions could be linear? Circle your answer(s). You do not have to show your work for this part.

$A(q)$ $\qquad$ $B(q)$ $\qquad$ $C(q)$ $\qquad$ $\boxed{D(q)}$

b. [3 points]
Which of the above functions could be exponential? Circle your answer(s). You do not have to show your work for this part.

$A(q)$ $\qquad$ $\boxed{B(q)}$ $\qquad$ $C(q)$ $\qquad$ $D(q)$

c. [4 points]
Find a possible formula for one of the functions above that you found could be linear. Show your work, and circle your answer.

Solution: The slope of $D(q)$ is $\frac{1}{6}$, so $D(q) = \frac{q}{6} + c$. Using the point $(4, 2)$, we see that $c = \frac{4}{3}$, so

$$D(q) = \frac{q}{6} + \frac{4}{3}$$

d. [5 points]
Find a possible formula for one of the functions above that you found could be exponential. Show your work, and circle your answer.

Solution: Using the last two columns of the table, we get that the growth factor for $B(q)$ is given by

$$\frac{27}{2} \cdot \frac{1}{9} = \frac{3}{2}$$

so that $B(q) = a(\frac{3}{2})^q$. Using the point $(1, \frac{8}{3})$, we get that

$$\frac{8}{3} = a(\frac{3}{2})$$

so that $a = \frac{16}{9}$, hence

$$B(q) = \frac{16}{9}(\frac{3}{2})^q$$

8. [12 points] Top Norwegian skier, Wallace, is participating in the ski jumping world championship. During his practice jump, his height (in meters) above his landing spot on the ground t seconds after going airborne is given by $h = A(t) = -4.9t^2 + 15t + 60$.

Throughout this problem include **units**, and express your answers in **exact form** or round your answers to **three decimal places**. Be sure show all the steps needed to get your answers, and **circle** your final answer. Answers with no work shown will not receive credit.

a. [2 points]
How high above his landing spot was Wallace when he first went airborne?

Solution: 60 meters

b. [4 points]
At what time t does Wallace land on the ground?

Solution: Using the quadratic formula, the roots are given by

$$t = \frac{-15 \pm \sqrt{(15)^2 + 4(4.9)(60)}}{-9.8}$$

One of the solutions is negative, hence doesn't make sense in the context of the problem. We throw that one out, so that the only correct solution is

$$t = \frac{-15 - \sqrt{(15)^2 + 4(4.9)(60)}}{-9.8} \approx 5.350$$

c. [6 points]
A daredevil skier, V, did a stunt jump with a jetpack on their back. Suppose that the quadratic function $B(t)$ gives V's height (in meters) above their landing spot t seconds after going airborne. From the reports, it was gathered that V's jump lasted 8 seconds, that they jumped from a height of 48 meters above ground, and that they reached maximum height 3 seconds after they went airborne. Find a formula for $B(t)$.

Solution:
If using vertex form: $B(t) = a(t-3)^2 + k$. Using the two intercepts, we have

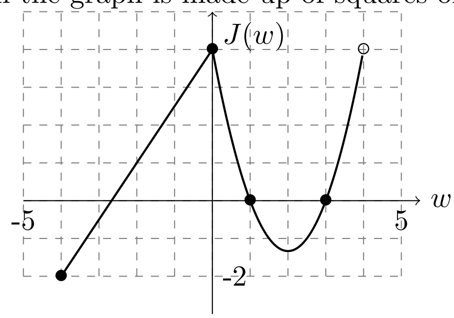
$$48 = a(9) + k$$

$$0 = a(25) + k$$

Solving the system of equations gives $a = -3$ and $k = 75$, so that $B(t) = -3(t-3)^2 + 75$.

Alternatively, one can use factored form. Knowing we have a root at $t = 8$, and using the symmetry of the parabola, the other root is at $t = -2$, so that $B(t) = c(t-8)(t+2)$. Using $B(0) = 48$, we get $c = -3$, so $B(t) = -3(t-8)(t+2)$.

9. [15 points] Below is a graph of a function $J(w)$ and a table of values for a function $T(z)$. The grid on the graph is made up of squares of side length one.



z	-3	-2	3	4	9
$T(z)$	9	3	1	3	c

a. [3 points]
Suppose the average rate of change of $T(z)$ between $z = -3$ and $z = 9$ is 2.5. Find c.

Solution:

$$\frac{c-9}{9-(-3)} = 2.5$$
$$c = 39$$

c= 39 .

b. [4 points]
Find all solutions to the equation

$$T(J(w)) = 3$$

using only the information about $J(w)$ and $T(z)$ above. Find exact answers if possible, or estimate using the grid if needed. Circle your final answer(s).

Solution: We look at which inputs of $T(z)$ are needed to get output 3. From the table, we need an input of -2 or 4, hence we need to look at what inputs of $J(w)$ give those outputs. From the graph, we see that we get two valid inputs

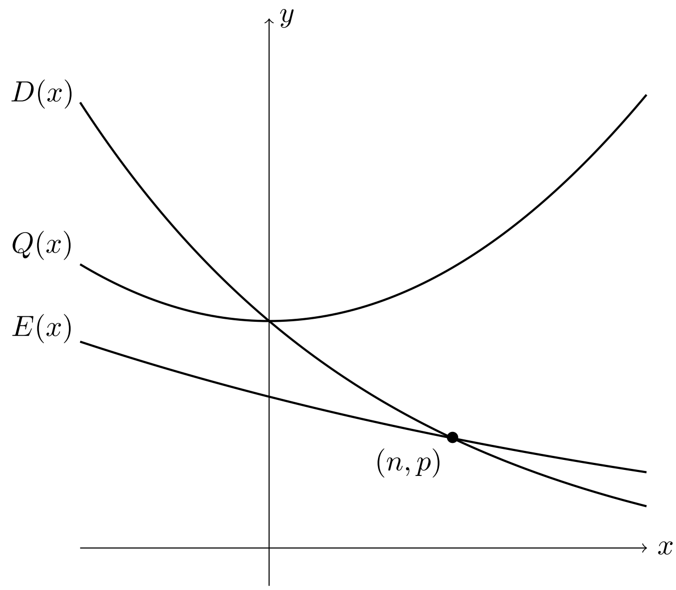
$$w = -4,\ 0$$

c. [8 points]
$J(w)$ is comprised of a linear piece and a quadratic piece. Find a piecewise-defined function for $J(w)$. Circle your answer.

Solution: The piecewise function consists of a linear part followed by a quadratic part. From the graph, the linear part goes between $(-4, -2)$ and $(0, 4)$, so we deduce that the linear function is $\frac{3}{2}w + 4$. For the quadratic part, we have two zeros at $w = 1$ and $w = 3$, so we put it into factored form $a(w-1)(w-3)$. Using the point $(0, 4)$, we deduce that $a = \frac{4}{3}$. Putting them together into the a piecewise function gives

$$J(w) = \begin{cases} \frac{3}{2}w + 4 & -4 \le w < 0 \\ \frac{4}{3}(w-1)(w-3) & 0 \le w < 4 \end{cases}$$

10. [8 points] The functions $D(x)$, $E(x)$, and $Q(x)$ are pictured below.



Suppose that

- $D(x) = d(1+r)^x$ is an exponential function.
- $E(x) = (1+h)^x$ is an exponential function.
- $Q(x) = ax^2 + c$ is a quadratic function.
- $D(x)$ and $E(x)$ intersect at the point (n, p).

In the formulas above, a, c, d, h, n, p, r are constants.

In each of the bullet points below, you are asked to circle the option that **must** be true based on the graph above. If there is **not enough information** to decide on any of the options in a given row, circle N/A.

- The constants r and h satisfy:

  $\boxed{r < h}$ $\quad$ $r > h$ $\quad$ $r = h$ $\quad$ N/A

- The constants c and d satisfy:

  $c < d$ $\quad$ $c > d$ $\quad$ $\boxed{c = d}$ $\quad$ N/A

- The constants a and h satisfy:

  $a < h$ $\quad$ $\boxed{a > h}$ $\quad$ $a = h$ $\quad$ N/A

- Suppose that we decrease the value of r. Then the value of n:

  Increases $\quad$ $\boxed{\text{Decreases}}$ $\quad$ Stays the Same $\quad$ N/A