

Math 105 — First Midterm — October 20, 2020

1. This exam has 4 pages including this cover. There are 6 problems. Note that the problems are not of equal difficulty, so you may want to skip over and return to a problem on which you are stuck.
 2. Please read the instructions for each individual problem carefully. One of the skills being tested on this exam is your ability to interpret mathematical questions, so instructors will not answer mathematical questions about exam problems during the exam.
 3. Show an appropriate amount of work (including appropriate explanation) for each problem, so that graders can see not only your answer but how you obtained it. Partial credit will be awarded for correct work.
 4. Problems may ask for answers in *exact form*. Recall that $x = \frac{1}{3}$ is an exact answer to the equation $3x = 1$, but $x = 0.333$ is not.
 5. You do not need to “simplify” your answers unless asked to do so.
 6. You must use the methods learned in this course to solve all problems. Logarithm functions taught in this course include “log” (log base 10) and “ln” (natural log).
 7. You may use one pre-written page of notes, on an 8.5”x11” standard sheet of paper, with whatever you want written on both sides.
 8. You will not be allowed to use any other resources, including calculators, other notes, or the book.
 9. You must write your work and answers on **blank, white, physical paper**.
 10. You must write your **initials and UMID**, but not your name or unickname, in the upper right corner of every page of work. Make sure that it is visible in all scans or images you submit.
 11. Make sure that all pages of work have the relevant problem number clearly identified.
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Problem	Points
1	2
2	8
3	12
4	9
5	13
6	13
Total	57

1. [2 points] **There is work to submit for this problem. Read it carefully.**

- You may use your one pre-written page of notes, on an 8.5" by 11" standard sheet of paper, with whatever you want written on both sides.
- You are not allowed to use any other resources, including calculators, other notes, or the book.
- You may not use any electronic device or the internet, except to access the Zoom meeting for the exam, to access the exam file itself, to submit your work, or to report technological problems via the Google forms we will provide to do so. The one exception is that you may use headphones (e.g. for white noise) if you prefer, though please note that you need to be able to hear when the end of the exam is called in the Zoom meeting.
- You may not use help from any other individuals (other students, tutors, online help forums, etc.), and may not communicate with any other person other about the exam until **8am on Wednesday** (Ann Arbor time).
- The one exception to the above policy is that you may contact the proctors in your exam room via the chat in Zoom if needed.
- Violation of any of the policies above may result in a score of zero for the exam, and, depending on the violation, may result in a failing grade in the course.

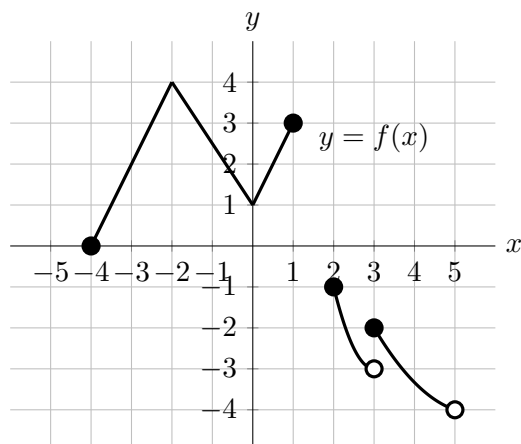
As your submission for this problem, you must write "I agree," and write your initials and UMID number to signify that you understand and agree to this policy. By doing this you are attesting that you have not violated this policy.

2. [8 points]

The entire graph of a function $y = f(x)$ is given to the right. Note that $f(x)$ is piecewise-linear for $-4 \leq x \leq 1$.

A different function, $g(x)$, is given in the following table.

x	-3	-2	-1	0	0.2	1	2	3
$g(x)$	5	4	1	0	-1	-2	-4	-3



a. [3 points] Find

- (i) the domain of $f(x)$ and
- (ii) the range of $f(x)$.

Write your answers using inequalities or interval notation. Make sure to clearly label which answer is the domain and which is the range. You do not need to justify your answer.

b. [5 points] Find the following values. If you make any calculations to find your answers, include those calculations in your submission.

- (i) The average rate of change of $f(x)$ from $x = -4$ to $x = 1$
- (ii) $g(f(3))$
- (iii) All values of x so that $g(f(x)) = -2$

3. [12 points] A gardener is growing a plant.

- Let t be the number of days after the plant first sprouts.
- The height of the plant t days after it sprouts is $H(t)$ inches.
- The gardener gives the plant $W(t)$ cups of water on the t^{th} day after it sprouts.
- When the gardener uses M cups of water, she mixes in $V(M)$ teaspoons of special plant vitamins.

Suppose that $V(M)$ and $H(t)$ have inverses.

For each of the following, give a practical interpretation of the expression in the context of the problem, or explain why the expression does not make sense in this context.

- a. [3 points] $H(6) = 3$
- b. [3 points] $V(W(4))$
- c. [3 points] $W(H(6))$
- d. [3 points] $\frac{H^{-1}(12) - H^{-1}(9)}{12 - 9} = 2$

4. [9 points] Let $h(w)$ be a function, with values given in the following table:

w	1	2	4
$h(w)$	8	10	14

- a. [2 points] Briefly explain why $h(w)$ could be a linear function of w , and find a linear formula for $h(w)$. For the rest of the problem, assume that $h(w)$ is linear.
- b. [3 points] Let $q(w)$ be a quadratic function of w , where $q(w)$ has its vertex at $(1, 8)$ and passes through the w -intercept of $h(w)$. Find a formula for $q(w)$.
- c. [1 point] Is the graph of $q(w)$ concave up or concave down?
- d. [3 points] Find the zeroes of $q(w)$.
5. [13 points] You're buying baking supplies (flour, butter, apples, etc.) for your pie business. Let $A(d)$ be the amount of money (in dollars) you'll get back from selling the pies you make from d dollars of supplies. For each dollar you spend on supplies between 0 and 50 dollars, you'll get back \$2. For each additional dollar over the first 50 dollars you invest you get back \$3.

- a. [4 points] From the description above, write down a piecewise formula for the function $A(d)$. Use standard piecewise notation:

$$A(d) = \left\{ \begin{array}{l} \\ \\ \\ \end{array} \right.$$

- b. [4 points] Evaluate $A^{-1}(190)$ and give a practical interpretation of your answer.
- c. [5 points] Find a piecewise formula for the composition $A(A(d))$. Use standard piecewise notation.
6. [13 points] You're looking at buying two cars:
- Car A is worth \$30,000 initially, and the value decreases by 15% annually.
 - Car B is worth \$20,000 initially, and the value also decreases exponentially. Let r be the annual growth rate. Note that r is negative.
- a. [3 points] If we know that the values of Car A and Car B will be equal at some point in the future, which of the following must be true? Briefly explain your reasoning.
- i. $r < -0.15$.
 - ii. $r > -0.15$.
 - iii. We do not have enough information to decide.
- b. [3 points] Suppose that r is some value so that the cars do eventually become the same price, and then r increases (so r gets closer to 0) and everything else stays the same. Will the time it takes for the two cars to become equal in value increase or decrease? Briefly explain your reasoning.
- c. [4 points] Let t be the number of years from now when the two cars are equal in value. Find t (in exact form). Your answer may contain r .
- d. [3 points] If the cars will be equal in value in 10 years, find r (in exact form).