Math 105 — First Midterm — October 12, 2021

EXAM SOLUTIONS

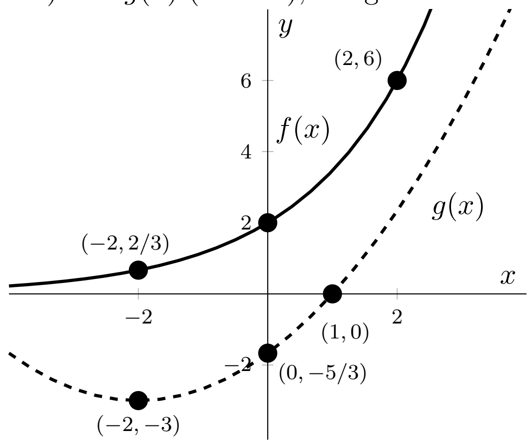
1. **Do not open this exam until you are told to do so.**
2. **Do not write your name anywhere on this exam.**
3. This exam has 0 pages including this cover. There are 0 problems.
   Note that the problems are not of equal difficulty, so you may want to skip over and return to a problem on which you are stuck.
4. Do not separate the pages of this exam. If they do become separated, write your UMID (not name) on every page and point this out to your instructor when you hand in the exam.
5. The back of every page of the exam is blank, and, if needed, you may use this space for scratchwork. Clearly identify any of this work that you would like to have graded.
6. Read the instructions for each individual problem carefully. One of the skills being tested on this exam is your ability to interpret mathematical questions, so instructors will not answer questions about exam problems during the exam.
7. Show an appropriate amount of work for each problem, so that graders can see not only your answer but how you obtained it.
8. You must use the methods learned in this course to solve all problems.
9. You are allowed notes written on two sides of a $3'' \times 5''$ note card and one calculator that does not have an internet or data connection (scientific or graphing recommended).
10. If you use a graph or table to find an answer, be sure to sketch the graph or write out the entries of the table. In either case, include an explanation of how the graph or table gives the answer.
11. Include units in your answer where that is appropriate.
12. Problems may ask for answers in *exact form.* Recall that $x = \sqrt{2}$ is a solution in exact form to the equation $x^2 = 2$, but $x = 1.41421356237$ is not.
13. **Turn off all cell phones, smartphones, and other electronic devices**, and remove all headphones, earbuds, and smartwatches. Put all of these items away. The use of any networked device while working on this exam is not permitted.

Problem	Points	Score
1		
2		
3		

Problem	Points	Score
Total		

1. [0 points]

Parts of the graphs of two functions, $f(x)$ (solid) and $g(x)$ (dashed), are given below.



The table below contains some information about the functions $p(x)$ and $q(x)$.

x	-1	0	1	2
$p(x)$	0	3	0	5
$q(x)$	0.8	-0.8	-2.4	-4

Exactly **one** of $f(x), g(x), p(x)$, and $q(x)$ is exponential, exactly one is linear, and exactly one is quadratic.

In the following questions, choose one option and then fill in the blanks with the correct values based on whichever function you chose.

a. [3 points] Which of the functions above is linear? Circle your answer.

$f(x)$ $g(x)$ $p(x)$ $\boxed{q(x)}$

Identify the slope and vertical intercept of the linear function.

Solution:

Slope: $\underline{-1.6}$ Vertical intercept: $\underline{-0.8}$

b. [4 points] Which of the functions above is exponential? Circle your answer.

$\boxed{f(x)}$ $g(x)$ $p(x)$ $q(x)$

Identify the initial value, growth factor, and growth rate of the exponential function.

Solution:

Initial Value: $\underline{2}$ Growth Factor: $\underline{\sqrt{3}}$ Growth Rate: $\underline{\sqrt{3}-1}$

c. [6 points] Which of the functions above is quadratic? Circle your answer.

$f(x)$ $\boxed{g(x)}$ $p(x)$ $q(x)$

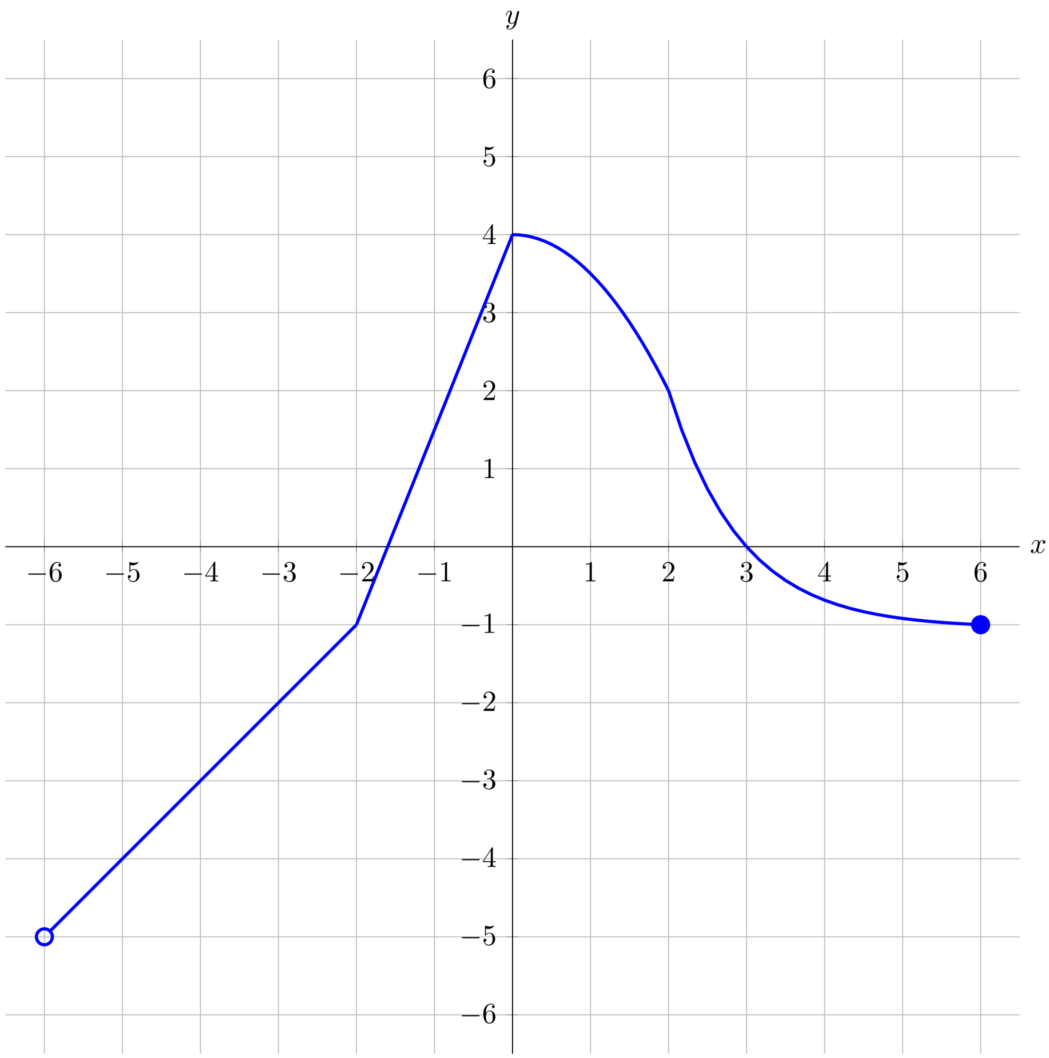
Find the zeros, the coordinates of the vertex, and a formula for the function.

Solution:

Zeros: $x = \underline{1 \text{ and } -4}$ Vertex: $(\underline{-2}, \underline{-3})$ Formula: $y = \underline{\frac{1}{3}(x+2)^2 - 3}$

2. [10 points] On the axes provided below, sketch the graph of **one** possible function $y = f(x)$, satisfying all of the following requirements. Your graph should clearly show the properties listed below to receive full credit.

- The domain of $f(x)$ is the interval $-6 < x < 6$.
- The range of $f(x)$ is $-5 < y \leq 4$.
- $f(-4) = -3$.
- $f(x)$ has a constant rate of change for $-6 < x < -3$.
- $f(x)$ is increasing on the interval $-3 < x < 0$.
- The average rate of change of $f(x)$ is -1 for $0 \leq x \leq 2$.
- $f(x)$ has a zero at $x = 3$.
- The graph of $f(x)$ is concave up for $2 < x < 6$.
- $f(x)$ is decreasing for $2 < x < 6$.



3. [0 points] The Smashing Squash are at an event signing autographs. They realized that they can model the length of the line waiting to get into the store as a function $J(t)$, where t is the number of hours after they arrived, using the following information.

- When they first arrived at the event the line was 12 feet long.
- The line then grew at a constant rate of 18 feet per hour until the line was 120 feet long. At this time, the venue stopped allowing people to come and so the line stopped growing in length.
- From that time on, the line decreased in length at a constant rate.
- Two hours after the line stopped growing, it was 98 feet long.
- The Squash left 14 hours after arriving. *Note that $J(t)$ is defined only while the band is at the event.*

a. [3 points] How many hours had the band been signing before the line stopped growing?

Solution:
The line's length after t hours is $12 + 18t$, so we want to know when $12 + 18t = 120$. We then solve for t.

Answer: **6** hours

b. [2 points] What is the domain of $J(t)$ in the context of this problem? Use either inequality or interval notation.

Solution: **Answer:** Domain: **[0,14]**

c. [6 points] Give a formula for $J(t)$ that is valid on its entire domain.

Solution:
We have the formula on the first part already, $12 + 18t$. For the second part, we find the slope between the points $(6, 120)$ and $(8, 98)$, which is -11. Then we use point-slope form with the point $(6, 120)$ for this piece.

Answer:

$$J(t) = \begin{cases} 12 + 18t & \text{for } 0 \leq t \leq 6 \\ 120 - 11(t-6) & \text{for } 6 < t \leq 14 \end{cases}$$

4. [0 points] Alex Artakis, the lead singer of Neverclear, left his pool uncovered when he went on tour. Due to the warm weather the pool loses 2.25% of its volume every week after the tour began. Let $v(w)$ be the volume, in m^3, of Alex's pool w weeks after the tour began. When he left to go on tour the pool was full and had a volume of 120 m^3.

a. [2 points] Based on the description above, answer each of the following questions. Pick the best answer for each – you do NOT need to explain your reasoning for this question.

(i) Which of the following accurately describes $v(w)$?

i. $v(w)$ is increasing

ii. $v(w)$ is decreasing

iii. $v(w)$ is constant

iv. NONE OF THESE

(ii) What kind of function is $v(w)$?

i. $v(w)$ is linear

ii. $v(w)$ is quadratic

iii. $v(w)$ is exponential

iv. NONE OF THESE

b. [4 points] Write a formula for $v(w)$ in terms of w, the number of weeks since the tour began.

Solution:
We know that the initial value is 120. Since it decreases by 2.25% each week, the growth factor is $1 - 0.0225 = 0.9775$.

Answer: $v(w) =$ $120\,(0.9775)^w$

c. [4 points] Evaluate $v(10)$, giving your answer in exact form or rounded to the nearest hundredth, and give a practical interpretation of your answer in the context of the problem. *Use a complete sentence and **include units**.*

Solution: **Answer:** $v(10) =$ $120\,(0.9775)^{10} \approx 95.58\mathbf{m^3}$.

Interpretation:

Solution: 10 weeks after the tour began there are approximately 95.58 cubic meters of water left in Alex's pool.

5. [0 points] At concerts put on by the band Emergency Kittens, the band tours with kittens that are available for adoption, and plays soothing music while concert-goers play with the kittens.

- $K = g(t)$ is the number of kittens traveling with the band t days into their tour.
- $S = h(K)$ is the amount of time, in hours per day, that band members spend snuggling with kittens when they are traveling with K kittens.
- $h^{-1}(S)$ is a function. (That is, $h(K)$ is invertible.)
- Some values of t and K are given in the table below.

t	3	5	8	9
K	18	22	23	22

a. [3 points] Based on the information in the table, could t be a function of K? Briefly explain your answer.

Answer *(circle one):*

Yes (t could be a function of K) **No** (t could **not** be a function of K)

Explanation:

Solution: If t were to be a function of K then the "input" $K = 22$ would have two outputs: $t = 5$ and $t = 8$. Thus, there is not one unique output for every input and we cannot have t as a function of K.

b. [4 points] Using the table, find the average rate of change of $g(t)$ from $t = 3$ to $t = 8$, and interpret your answer in the context of the problem.

Solution: We use

$$\text{average rate of change} = \frac{K(8) - K(3)}{8 - 3} = \frac{5}{5} = 1.$$

Answer: **1 kitten/day**

Interpretation:
From day 3 of the tour to day 8 of the tour the band increases the number of kittens traveling with them by an average of 1 kitten per day.

c. [9 points] For each of the following, either give a practical interpretation of the mathematical expression, or explain why it doesn't make sense in the context of the problem.

(i) $g(10) = 25$

Solution: 10 days into the tour the band had 25 kittens traveling with them.

(ii) $h(g(4))$

Solution: $h(g(4))$ is the amount of time the band spends snuggling with kittens when they have been on tour for 4 days.

(iii) $h^{-1}(5) \geq 8$

Solution: When the band spends 5 hours per day snuggling with kittens, they are traveling with at least 8 kittens.