Math 105 — Second Midterm — November 17, 2022

1. **Do not open this exam until you are told to do so.**
2. **Do not write your name anywhere on this exam.**
3. This exam has 8 pages including this cover. There are 7 problems.
   Note that the problems are not of equal difficulty, so you may want to skip over and return to a problem on which you are stuck.
4. Do not separate the pages of this exam. If they do become separated, write your UMID (not name) on every page and point this out to your instructor when you hand in the exam.
5. The back of every page of the exam is blank, and, if needed, you may use this space for scratchwork. Clearly identify any of this work that you would like to have graded.
6. Read the instructions for each individual problem carefully. One of the skills being tested on this exam is your ability to interpret mathematical questions, so instructors will not answer questions about exam problems during the exam.
7. Show an appropriate amount of work for each problem, so that graders can see not only your answer but how you obtained it.
8. You must use the methods learned in this course to solve all problems.
9. You are allowed notes written on two sides of a $3'' \times 5''$ note card and one calculator that does not have an internet or data connection (scientific or graphing recommended).
10. If you use a graph or table to find an answer, be sure to sketch the graph or write out the entries of the table. In either case, include an explanation of how the graph or table gives the answer.
11. Include units in your answer where that is appropriate.
12. Problems may ask for answers in *exact form*. Recall that $x = \sqrt{2}$ is a solution in exact form to the equation $x^2 = 2$, but $x = 1.41421356237$ is <u>not</u>.
13. **Turn off all cell phones, smartphones, and other electronic devices**, and remove all headphones, earbuds, and smartwatches. Put all of these items away. The use of any networked device while working on this exam is <u>not</u> permitted.

Problem	Points	Score
1	9	
2	4	
3	11	
4	7	

Problem	Points	Score
5	5	
6	11	
7	13	
Total	60	

1. [9 points] Consider the table of known values for the functions $f(x)$ and $h(x)$, where $f(x)$ is invertible.

x	-4	-2	-1	0	1	2	4
$f(x)$	2	3	0	-2	-1	4	5
$h(x)$	$?$	2	1	4	0	$?$	7

a. [4 points] Find each of the following, or write N/A if a value does not exist or there is not enough information to find it.

(i) $f^{-1}(0)$ **Answer:** $f^{-1}(0) =$ _____

(ii) $f(h(0))$ **Answer:** $f(h(0)) =$ _____

(iii) $h(g(1))$, where $g(x) = \log(x)$

Answer: $h(g(1)) =$ _____

(iv) $k(1)$, where $k(x) = -4f(2(x+1)) - 6$

Answer: $k(1) =$ _____

b. [2 points] If $f(h(2)) = 0$, then what is $h(2)$?

Answer: $h(2) =$ _____

c. [3 points] Give a value for $h(-4)$ that would guarantee that $h(x)$ is *not* invertible and explain (in at most 1 sentence) why your value for $h(-4)$ forces the function to be non-invertible.

Answer: $h(-4) =$ _____

Explanation:

2. [4 points] Use the graph of $y = 10^x$ below to decide whether each of the following statements is true (T), false (F), or there is not enough information to tell (NEI).

y

$(a, 12)$

$y = f(x) = 10^x$

$(b, 0.02)$

x
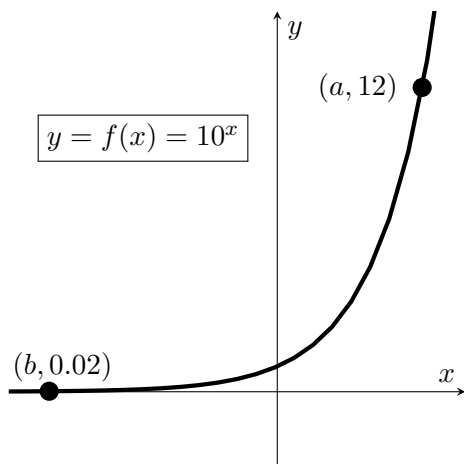
(i) $a < 1$ T   F   NEI

(ii) $b < -1$ T   F   NEI

(iii) $\log(12) = a$ T   F   NEI

(iv) $\log(b) = 0.02$ T   F   NEI

3. [11 points] Three bacteria colonies, called A, B, and C, are established at the same time.
The number of bacteria in these colonies are given by $A(t)$, $B(t)$, and $C(t)$, where t is
measured in hours since the colonies were established. The formulas for these functions are

$$A(t) = 200 \cdot 2^t$$
$$B(t) = 500 \cdot 3^{t+1}$$
$$C(t) = 100 \cdot e^{2t}$$

a. [1 point] How many bacteria did Colony B start with?

Answer: _____

b. [1 point] Which, if any, colonies have a percent growth rate of 200% per hour?
Circle **all** that are correct.

A   B   C   None

c. [6 points]
(i) Starting from the time the colonies were established, will colonies A and B ever have
the same number of bacteria? If so, find the time when this happens, in exact form
or rounded to at least two decimal places. If not, briefly explain why not.

Answer *(circle one):*   Yes: $t =$_____   No (explain below)

(ii) Starting from the time the colonies were established, will colonies A and C ever have
the same number of bacteria? If so, find the time when this happens, in exact form
or rounded to at least two decimal places. If not, briefly explain why not.

Answer *(circle one):*   Yes: $t =$_____   No (explain below)

Recall: A bacteria colony C has population $C(t)$, where t is measured in hours since the colony was established. The formulas for this function is

$$C(t) = 100 \cdot e^{2t}$$

d. [3 points] Find a formula for $g(P)$, a function that gives the amount of time (in hours) it takes for colony C to reach P bacteria.

Answer: $g(P) = $ _____

4. [7 points] Let $g(x) = 2 \cdot (0.5)^{-3x} - 6$.

a. [5 points] List the transformations you need to apply to the graph of $y = 0.5^x$ to transform it to that of $y = g(x)$. Fill each space with either a number or one of the phrases below, as appropriate. (Leave the second blank empty for reflections.)

SHIFT IT TO THE LEFT   STRETCH IT HORIZONTALLY   REFLECT IT ACROSS THE y-AXIS

SHIFT IT TO THE RIGHT   COMPRESS IT HORIZONTALLY   REFLECT IT ACROSS THE x-AXIS

SHIFT IT UP   STRETCH IT VERTICALLY

SHIFT IT DOWN   COMPRESS IT VERTICALLY

First, _____ by _____

then, _____ by _____

then, _____ by _____

then, _____ by _____

b. [2 points] Give equations for all vertical and horizontal asymptotes of $g(x)$. If there are none, write None.

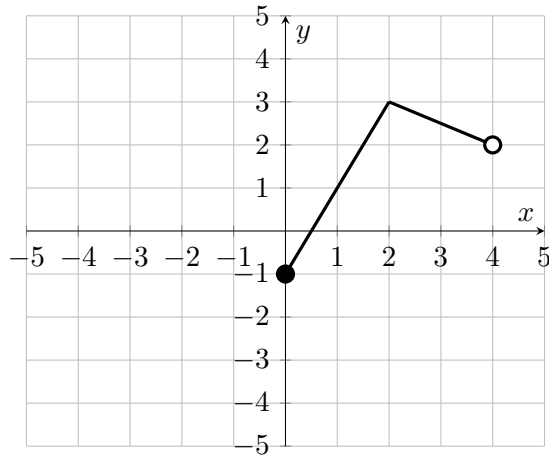**Answer:** Vertical Asymptotes: _____

Answer: Horizontal Asymptotes: _____

5. [5 points]

    a. On the axes below, part of the graph of a function $y = f(x)$ is given. Either draw in the rest of the graph to make the function **even**, or briefly explain why this is not possible.

Answer *(Circle one)*:   POSSIBLE   NOT POSSIBLE

Explanation:

    b. On the axes below, part of the graph of a function $y = g(x)$ is given. Either draw in the rest of the graph to make the function **odd**, or briefly explain why this is not possible.
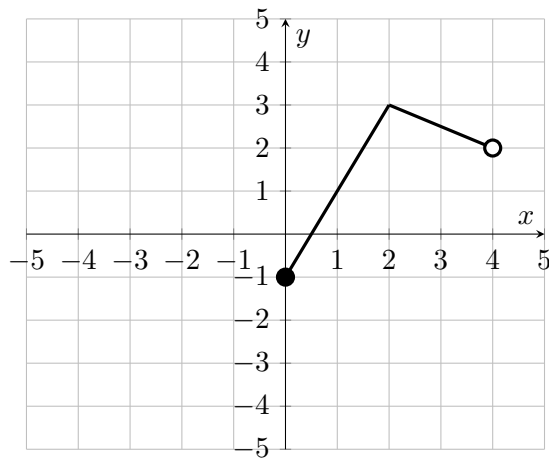
Answer *(Circle one)*:   POSSIBLE   NOT POSSIBLE

Explanation:

6. [11 points] A video is posted online and later goes viral after it is shared by a certain celebrity on a social media platform. 2 hours after it is shared, it has 5 thousand views, and 6 hours after it is shared, it has 10 thousand views.

   a. [2 points] Suppose that the number of views increases at a constant rate of views per hour. Find a formula for $f(t)$, the number of views, in thousands, that the video has t hours after it is shared.

   Answer: $f(t) =$ _____

   b. [4 points] Suppose instead that the number of views increases at a constant **percent** growth rate, find a formula for $g(t)$, the number of views, in thousands, that the video has t hours after it is shared.

   Answer: $g(t) =$ _____

   c. [2 points] Suppose that the number of views increases at a constant percent growth rate and M is a number greater than 4. Which of the following numbers is **greater**?

   - Let A be the time, in hours, it takes for the number of views to increase from 4 thousand to 12 thousand.
   - Let B be the time, in hours, it takes for the number of views to increase from M thousand to $3M$ thousand.

   Answer *(Circle one)*:

   A is greater B is greater   They are equal   Cannot be determined

   d. [3 points] Another video has gone viral, and the number of views for that video increases by 40% in 2 hours. Find the **continuous** hourly percent growth rate of the number of views of this video. Give your answer in exact form or correct to at least two decimal places.

   Answer: Continuous hourly percent growth rate: _____%

7. [13 points] Jada is riding on a train to visit Emerald Beach. Her friend Eva is also traveling to Emerald Beach, but she left on an earlier train. Eva has been tracking her remaining distance to Emerald Beach in sprites, which is the elves' measurement for distance.

- The function $E(m)$ gives Eva's distance away from Emerald Beach, in sprites, m minutes after Eva's train departed.
- Jada's train left 30 minutes after Eva's train.
- When Jada's train departs, it is 25 sprites behind Eva's train.
- At any given moment, both trains are traveling at the same speed, so Jada's train is always exactly 25 sprites farther away from Emerald Beach than Eva's train.

You do not need to show work for this problem.

a. [2 points] Let $J(t)$ be Jada's distance from Emerald Beach, in sprites, t minutes after *Jada's* train departed. Find a formula for $J(t)$ in terms of the function E and the variable t.

Answer: $J(t) =$ _____

b. [2 points] Give a practical interpretation of the following equation:

$$E(90) = \frac{1}{2}J(0)$$

Interpretation:

c. [5 points] Jada prefers to use earthly measurements of distance, and she knows that one sprite is exactly three miles.
Based on the given information, fill in the following blanks:

When Jada has been traveling for 2 **hours**, Eva has been traveling for _____ minutes.

When Eva is 15 sprites from Emerald Beach, Jada is _____ miles from Emerald Beach.

Let $D(h)$ be Jada's distance from Emerald Beach, in **miles**, h **hours** after *Jada's train* departed. Find a formula for $D(h)$ in terms of the function E and the variable h.

Answer: $D(h) =$ _____

d. [4 points] Some points of the function $E(m)$ are given in the table below. Using these points, find the coordinates of points that must be on the graph of $D(h)$.

m	30	105	150
$E(m)$	185	80	15

h			
$D(h)$			