

Math 105 — Final Exam — December 14, 2022

EXAM SOLUTIONS

1. **Do not open this exam until you are told to do so.**
2. **Do not write your name anywhere on this exam.**
3. This exam has 9 pages including this cover. There are 6 problems.
Note that the problems are not of equal difficulty, so you may want to skip over and return to a problem on which you are stuck.
4. Do not separate the pages of this exam. If they do become separated, write your UMID (not name) on every page and point this out to your instructor when you hand in the exam.
5. The back of every page of the exam is blank, and, if needed, you may use this space for scratchwork. Clearly identify any of this work that you would like to have graded.
6. Read the instructions for each individual problem carefully. One of the skills being tested on this exam is your ability to interpret mathematical questions, so instructors will not answer questions about exam problems during the exam.
7. Show an appropriate amount of work for each problem, so that graders can see not only your answer but how you obtained it.
8. You must use the methods learned in this course to solve all problems.
9. You are allowed notes written on two sides of a $3'' \times 5''$ note card and one scientific calculator (without graphing capabilities).
10. If you use a graph or table to find an answer, be sure to sketch the graph or write out the entries of the table. In either case, include an explanation of how the graph or table gives the answer.
11. Include units in your answer where that is appropriate.
12. Problems may ask for answers in *exact form*. Recall that $x = \sqrt{2}$ is a solution in exact form to the equation $x^2 = 2$, but $x = 1.41421356237$ is not.
13. **Turn off all cell phones, smartphones, and other electronic devices**, and remove all headphones, earbuds, and smartwatches. Put all of these items away. The use of any networked device while working on this exam is not permitted.

Problem	Points	Score
1	4	
2	10	
3	13	
4	11	

Problem	Points	Score
5	10	
6	12	
Total	60	
Mastery 6	5	

1. [4 points] A small candy company called Wonky Bars sells two types of candy: chocolate bars and gummy worms. They have found that the number of chocolate bars sold, C , is inversely proportional to the square of g , the number of bags of gummy worms sold. At one point, Wonky Bars sold 150 chocolate bars and 30 bags of gummy worms. Give a formula for $C = f(g)$, the number of chocolate bars Wonky Bars sold when they sold g bags of gummy worms. *You do not need to simplify your answer.*

Solution: Since C is inversely proportional to the square of g , we have

$$C = k/g^2$$

for some value of k . Now we plug in the point we've been given and solve for k : $150 = k/30^2$, so $k = 150 \cdot 30^2$.

Answer: $f(g) = \underline{\hspace{10em} 150 \cdot 30^2/g^2 \hspace{10em}}$

2. [10 points] The parts of this problem are not related.

- a. [2 points] If $p(x)$ is a polynomial of degree 5 such that $\lim_{x \rightarrow \infty} p(x) = \infty$ and $\lim_{x \rightarrow -\infty} p(x) = -\infty$, which of the following are possible leading terms of $p(x)$?

Circle all correct options.

$5x^3$ $-5x^3$ $-\frac{1}{2}x^5$ $3x^5$ $-2x^5$ $\frac{3}{4}x^5$ None of these

- b. [2 points] If $q(x)$ is a polynomial such that $\lim_{x \rightarrow \infty} q(x) = -\infty$ and $\lim_{x \rightarrow -\infty} q(x) = \infty$, which of the following are possible degrees of $q(x)$?

Circle all correct options.

5 600 -1 755 2 None of these

- c. [2 points] Which of the following functions approach 0 as $x \rightarrow -\infty$?

Circle all correct options.

$\frac{x^2 + 8x^3}{x(6+x)(2x-1)}$ $\frac{e^x + 3}{x^2 + 1}$ $\frac{2^x + 5}{3^x + 7}$ $\frac{1}{\ln(-x)}$ None of these

- d. [2 points] Which of the following functions dominates all the others as $x \rightarrow \infty$?

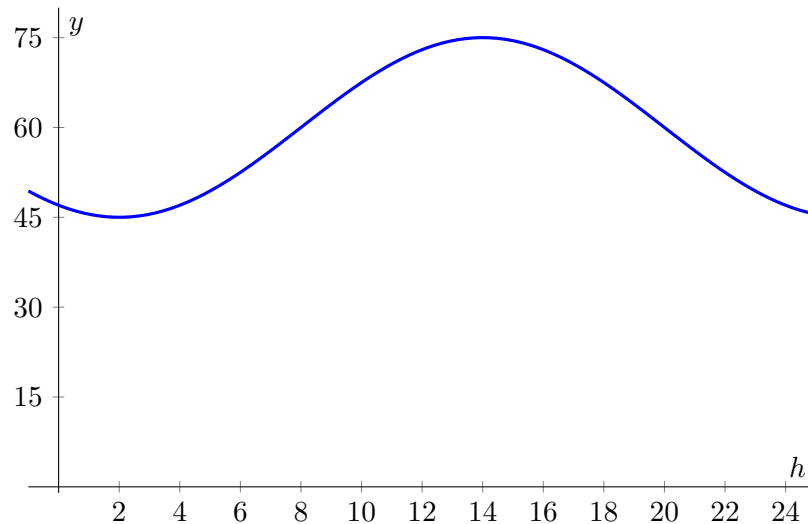
Circle exactly **one** of the options.

$100x + 650$ $5e^x$ $2(3)^x$ $2(3)^{-x}$ $15x^4 + x + 6$ $75x^{500}$

- e. [2 points] If θ is an angle with $\cos(\theta) = a$ for some positive number a , which of the following values must also equal a ? Circle all correct options.

$\cos(-\theta)$ $\cos(\pi - \theta)$ $\cos(2\pi - \theta)$ $\cos(\pi + \theta)$ None of these

3. [13 points] Jada remembers from her time in Dreamland that the temperature was very consistent every day: It would increase from a low of 45° Fahrenheit at 2 am to a high of 75° Fahrenheit at 2 pm. The temperature, in degrees Fahrenheit, h hours after midnight could be modeled by a sinusoidal function $T(h)$. Dreamland days are 24 hours.
- a. [4 points] On the axes below, sketch a graph of $y = T(h)$, showing at least one full period. *Clearly label the axes and important points on your graph. Be very careful with the shape and key features of your graph.*



- b. [5 points] Find a sinusoidal formula for $T(h)$. You do not need to show work.

Answer: $T(h) = \underline{\hspace{10em} -15 \cos\left(\frac{2\pi}{24}(h - 2)\right) + 60 \hspace{10em}}$

- c. [4 points] Jada's elf friend, Alf, ran an apple stand, and found that the length of the line for his stand, in meters, could be modeled by an invertible function $g(F)$, where F is the current temperature in degrees Fahrenheit. Interpret the meaning of the following mathematical expressions or equations, or explain why they don't make sense in the context of the problem.

(i) $g^{-1}(20)$

Solution: $g^{-1}(20)$ is the temperature, in degrees Fahrenheit, when the line is 20 meters long.

(ii) $g(T(14)) = 8$.

Solution: At 2 pm, the line for Alf's apple stand is 8 meters long.

4. [11 points] Dahlia is running an experiment. A weight bobs up and down on the end of a spring that is attached to the ceiling. The distance, in inches, between the spring and the ceiling t seconds after Dahlia begins recording is given by

$$h(t) = 3 \sin\left(\frac{4\pi}{3}t\right) + 8$$

- a. [3 points] Find the period of $h(t)$, and interpret your answer in the context of the problem.

Answer: Period: 3/2

Interpretation:

Solution: From one time the weight reaches its maximum distance to the next, 1.5 seconds pass.

- b. [2 points] Which of the following best describes the spring's motion at the moment Dahlia begins recording? Choose the **one** best answer.

When Dahlia begins recording...

- i. the spring is at its average distance from the ceiling, and is moving away from the ceiling.
 - ii. the spring is at its average distance from the ceiling, and is moving toward the ceiling.
 - iii. the spring is at its farthest point from the ceiling.
 - iv. the spring is at its closest point to the ceiling.
 - v. none of these
- c. [6 points] Find the first **two** times the spring is exactly 9 inches from the ceiling. Show all of your work, and give your answers in exact form or correct to at least two decimal places. Include units.

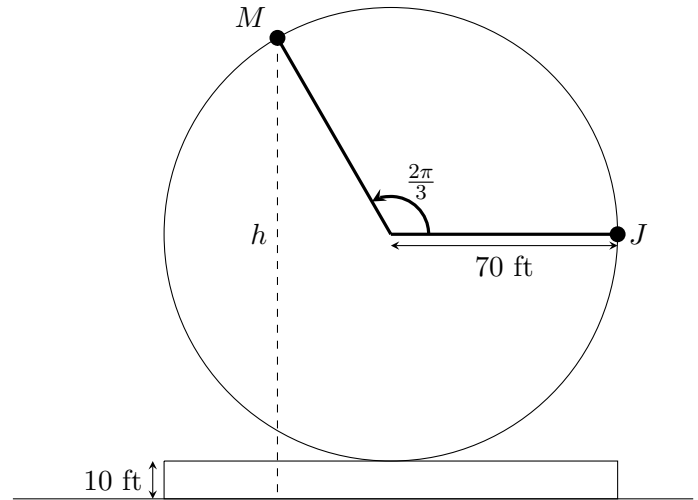
Solution: We find one solution by solving using arcsin:

$$\begin{aligned} 3 \sin\left(\frac{4\pi}{3}t\right) + 8 &= 9 \\ 3 \sin\left(\frac{4\pi}{3}t\right) &= 1 \\ \sin\left(\frac{4\pi}{3}t\right) &= 1/3 \\ \frac{4\pi}{3}t &= \arcsin(1/3) \\ t &= \frac{3}{4\pi} \arcsin(1/3) \end{aligned}$$

Since the period is $3/2$, the second solution is $3/4 - \frac{3}{4\pi} \arcsin(1/3)$

5. [10 points]

Jack takes his little sibling Mo on a Ferris wheel. The Ferris wheel has a radius of 70 feet, and the passengers board on a platform that is 10 feet above the ground. Seats on the Ferris wheel only take one rider at a time, so Jack and Mo end up at different positions on the Ferris wheel. The diagram to the right depicts the moment when Jack is at point J on the Ferris wheel, which is midway between the bottom and the top, and Mo is at point M . The angle between J and M is $\frac{2\pi}{3}$ radians. The Ferris wheel moves counterclockwise.



You do not need to show work for this problem, but may receive partial credit for correct work shown. Give your answers in exact form or correct to at least two decimal places.

- a. [2 points] Assuming the wheel rotates at a constant speed, Jack will reach Mo's current position on the Ferris wheel in exactly 5 minutes. How long does it take the Ferris wheel to make one complete revolution? **Include units.**

Answer: _____ **15 min** _____

- b. [2 points] How far did Mo travel along the circumference of the Ferris wheel between point J and point M ? **Include units.**

Answer: _____ **$140\pi/3 \approx 146.61$ (or **146.60**) ft** _____

- c. [3 points] Find the value of h , Mo's current height above the ground, in feet.

Answer: _____ **$10 + 70 \sin(2\pi/3) = 10 + 35\sqrt{3} \approx 70.62$** feet

- d. [3 points] Find the **horizontal distance** between Jack at point J and Mo and point M , in feet.

Answer: _____ **$70 - 70 \cos(2\pi/3) = 70 + 35 = 105$** feet

6. [12 points] The functions $P(x)$ and $Q(x)$ below are two polynomials.

$$P(x) = 5(3x^2 - 4)^2(2x + 3)(x - 1)$$

$$Q(x) = 5x(4x - 4)(2x + 3)^2$$

Let $R(x)$ be the rational function given by $R(x) = \frac{P(x)}{Q(x)}$.

Find the following. If there is no answer for a given question, write NONE. You do not need to show work.

- a. Find all zeros of $P(x)$.

Answer: Zeros at $x =$ $\pm\sqrt{4/3}, 1, -3/2$

- b. Find all zeros of $Q(x)$.

Answer: Zeros at $x =$ $0, 1, -3/2$

- c. Find all zeros of $R(x)$.

Answer: Zeros at $x =$ $\pm\sqrt{4/3}$

- d. Give the **equations** of any vertical asymptote(s) of $R(x)$.

Answer: $x = 0$ and $x = -3/2$

- e. Give the **equations** of any horizontal asymptote(s) of $R(x)$.

Answer: none

- f. Give the (x, y) coordinates of any holes of $R(x)$.

Answer: $(1, 1/20)$

Mastery 6 problems

If you are satisfied with your score on Mastery 6, you do not need to do these problems. You do not need to show work for these problems.

1. Find a formula for a power function $p(x)$, where $p(3) = 5$ and $p(7) = 6$.

Solution:

$$\frac{5}{3^{\ln(6/5)/\ln(7/3)}} x^{\ln(6/5)/\ln(7/3)}$$

Answer: $p(x) =$ _____

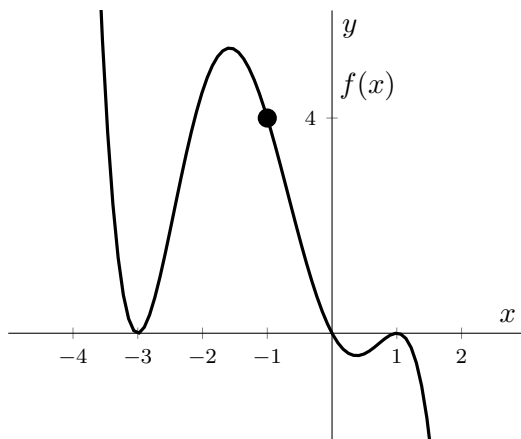
2. For each of the following, give the correct value. Write NONE if the mathematical sentence cannot be completed with a number or $\pm\infty$.

(i) $\lim_{x \rightarrow \infty} \frac{100x^3 - 50x^4}{-9x^4 + 5x^5} =$ _____ 0 _____

(ii) $\lim_{x \rightarrow -\infty} (10x^6 - 5x^4) =$ _____ ∞ _____

(iii) $\lim_{x \rightarrow \infty} 83x^{3.5} =$ _____ ∞ _____

3. Find a formula for the polynomial $p(x)$ of least degree that is graphed below. The polynomial has x -intercepts at $x = -3, 0$ and 1 , and passes through the point $(-1, 4)$.



Answer: $p(x) =$ _____ $(-1/4)(x+3)^2(x-1)^2x$ _____

Mastery 6 problems

If you are satisfied with your score on Mastery 6, you do not need to do these problems. You do not need to show work for these problems.

4. For each of the following functions, circle all options that apply.

(i) The function $f(u) = \frac{6}{u}$ is:

- A. polynomial
- B. power function
- C. neither a polynomial nor a power function

(ii) The function $g(s) = -\frac{5}{36}s^3 + 8s^{15}$ is:

- A. polynomial
- B. power function
- C. neither a polynomial nor a power function

(iii) The function $j(t) = \frac{3}{25}t^5$ is:

- A. polynomial
- B. power function
- C. neither a polynomial nor a power function

(iv) The function $k(x) = \frac{1}{9} + 9x^{1/7}$ is:

- A. polynomial
- B. power function
- C. neither a polynomial nor a power function

5. For the rational function $y = -\frac{3(t+4)(t+2)(t+3)}{(t-4)t^2}$ list the t -values of all vertical asymptotes, and of all zeros. If there are no asymptotes, or no zeros, enter NONE for that answer.

Answer: Vertical asymptotes: $t =$ 4, 0

Answer: Zeros: $t =$ -4, -2, -3