Math 105 — Second Midterm — November 14, 2023

Write your 8-digit UMID number very clearly in the box to the right, and fill out the information on the lines below.

Your Initials Only: ______ Your 8-digit UMID number (not uniqname): ____________________

Instructor Name: ____ Section #: __________

1. **Do not open this exam until you are told to do so.**
2. **Do not write your name anywhere on this exam.**
3. Use a pencil for "bubble-in" questions so that you can easily erase your answer if you change your mind.
4. This exam has 9 pages including this cover. There are 6 problems. Note that the problems are not of equal difficulty, so you may want to skip over and return to a problem on which you are stuck.
5. Do not separate the pages of this exam. If they do become separated, write your UMID (not name) on every page and point this out to your instructor when you hand in the exam.
6. The back of every page of the exam is blank, and, if needed, you may use this space for scratch-work. Clearly identify any of this work that you would like to have graded.
7. Read the instructions for each individual problem carefully. One of the skills being tested on this exam is your ability to interpret mathematical questions, so while you may ask for clarification if needed, instructors are generally unable to answer such questions during the exam.
8. Show an appropriate amount of work for each problem, so that graders can see not only your answer but how you obtained it.
9. You must use the methods learned in this course to solve all problems.
10. You are allowed notes written on two sides of a $3'' \times 5''$ note card and one scientific calculator that does not have graphing or internet capabilities.
11. Include units in your answer where that is appropriate.
12. Problems may ask for answers in *exact form* or in *decimal form*. Recall that $\sqrt{2} + \cos(3)$ is in exact form and 0.424 would be the same answer expressed in decimal form.
13. **Turn off all cell phones, smartphones, and other electronic devices**, and remove all headphones, earbuds, and smartwatches. Put all of these items away. The use of any networked device while working on this exam is <u>not</u> permitted.

Problem	Points	Score
1	9	
2	6	
3	15	

Problem	Points	Score
4	7	
5	13	
6	10	
Total	60	

1. [9 points] The function $h(r)$ is **even** and **periodic** with period 6. Some values of $h(r)$ are given in the table below.

r	-2	0	1	3
$h(r)$	1.5	1	0	-1

The function $f(x)$ is given by $f(x) = \log(3x)$.

a. [5 points] Evaluate each of the following using information you know about $h(r)$; or if there is not enough information to so so, write NEI. *Write your answer in exact form, or rounded to two decimal places. Where relevant, show all work.*

(i.) $h(2) =$ ____________

(ii.) $h(h(0)) =$ ____________

(iii.) $h(10) =$ ____________

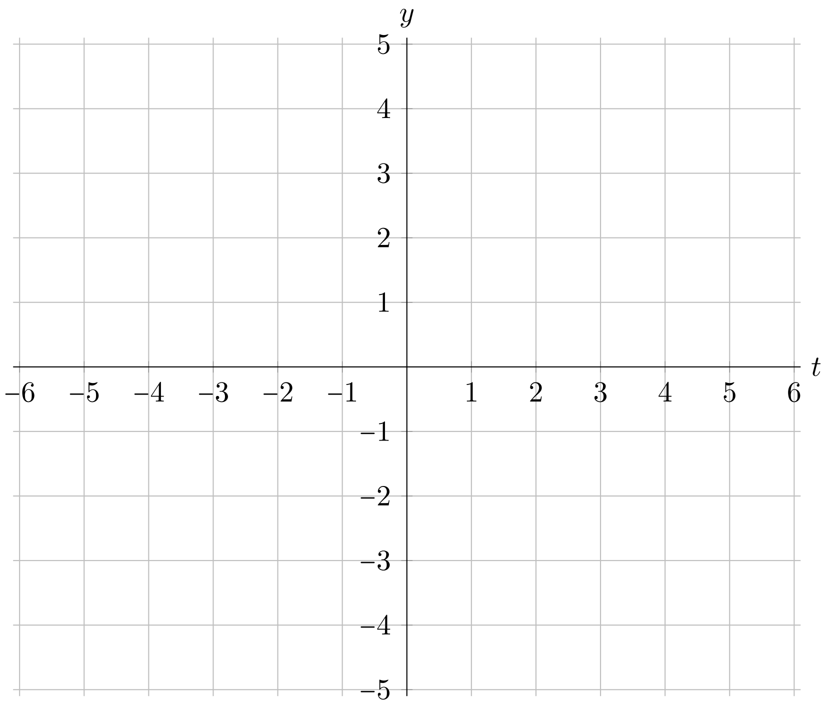
(iv.) $f^{-1}(h(-2)) =$ ____________

b. [2 points] Is $h(t)$ invertible? Explain why it definitely is, why it definitely isn't, or if there isn't enough information to tell.

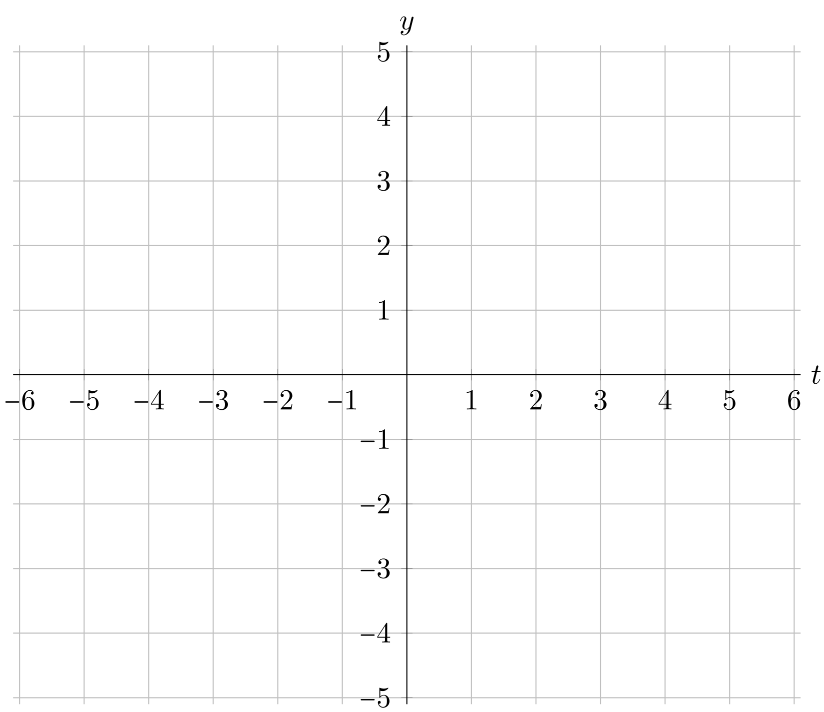
c. [2 points] Without using a calculator, Laurel claims that $f(33)$ is approximately 2. Explain how she could have known this!

2. [6 points] For each of the following, sketch a graph of a function meeting the stated criteria, or explain why no such function exists.

a. [3 points] An odd, invertible function defined on the domain $[-6, 6]$



b. [3 points] A periodic function with period 4, amplitude 3, and midline $y = -1$. *Include at least three periods in your sketch.*



3. [15 points] A scientist is observing two different ant colonies under different experimental conditions. From her data, it looks like

- Colony A's population increases by 10% every two hours.
- Colony B's population decreases by 7% every hour.

a. [1 point] By what factor is Colony A's population multiplied each hour? *Give your answer in exact form or rounded to two decimal places.*

a factor of: ____________

b. [2 points] What is the *continuous* decay rate of Colony B per hour as a percentage? *Give your answer in exact form or rounded to two decimal places.*

__________________ %

c. [2 points] How long will it take for Colony B to reach 25% of its original size? *Show all work. Give your answer in exact form or rounded to two decimal places.*

____________ hours

d. [4 points] If Colony A starts with 1000 ants and Colony B starts with 10,000 ants, after how many hours will the colonies have equal populations? *Show all work. Give your answer in exact form or rounded to two decimal places.*

____________ hours

(Problem continues on the next page.)

The scientist now observes two additional different ant colonies. From her data, it looks like

- Colony C's population doubles for the first time after 2.5 hours; doubles again 1.5 hours after that; then doubles a third time 1 hour after that.
- Colony D's population is given by $P = D(t) = 1200 - 300e^{-0.11t}$, where P is the number of ants and t is measured in hours since the experiment started.

e. [2 points] Is Colony C growing exponentially? Circle your answer below. If YES, find its growth factor. If NO, explain why not.

YES NO

Explanation or Growth Factor:

f. [4 points] Find a general formula $D^{-1}(P)$ and explain what that function means. *Show all work.*

$D^{-1}(P) =$ ____________________

Meaning of $D^{-1}(P)$:

4. [7 points] On a warm fall day, Schinella decides to walk home from work. Let $d = f(t)$ be the function giving Schinella's distance **from work**, in miles, t minutes after she leaves work.

a. [3 points] Her walk home from work is 3 miles. Schinella wants to write a new function $g(h)$ that gives her distance **from home**, in miles, h **hours** after she leaves work. Write a formula for $g(h)$ in terms of f.

$g(h) =$ ______________________

b. [2 points] Schinella (who is from Canada) wants to write another new function $k(t)$ that gives her distance from work in **kilometers** t minutes after she leaves work. Given that 1 mile is about 1.6 kilometers, circle the correct formula for $k(t)$ below.

$1.6f(t)$ $\qquad$ $f(1.6t)$ $\qquad$ $\dfrac{1}{1.6}f(t)$ $\qquad$ $f\left(\dfrac{t}{1.6}\right)$

c. [2 points] Let $c(t)$ be the function that gives the number of episodes of the podcast *Canadaland* that Schinella has listened to in the first t minutes of her walk. Assume that both $c(t)$ and $f(t)$ are invertible. Using those functions or their inverses, write an expression for Schinella's distance from work, in miles, after she's listened to 2.5 episodes of *Canadaland* while walking home.

______________________ miles

5. [13 points]

a. [4 points] A zookeeper has determined that the function $w(t)$ below provides a good model of the weight, in ounces, of a certain kind of snake t years after it hatches.

$$w(t) = -2e^{-(t-16)/5} + 52$$

Find the value of each of the following **as numbers rounded to two decimal places**. Then **briefly interpret** what each quantity means in the context of the problem.

i. $w(0) =$ ____________ **Meaning:**

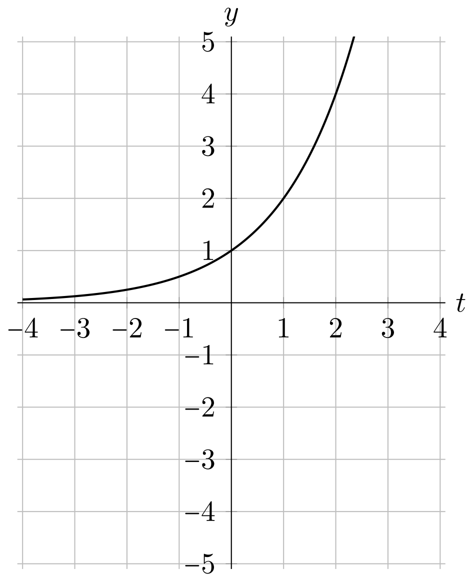
ii. $\lim_{t\to\infty} w(t) =$ ____________ **Meaning:**

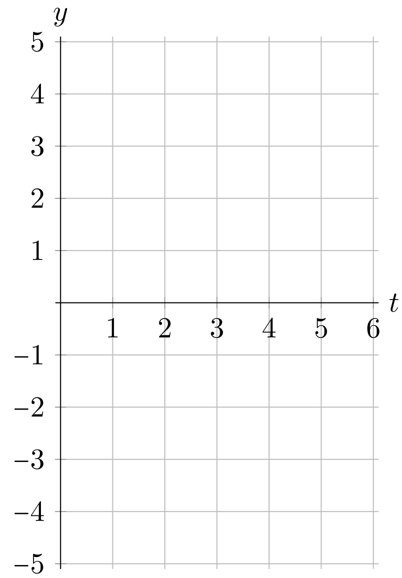
(Problem continues on the next page.)

b. [2 points] The zookeeper also has a model $\ell(t)$ of the length, in feet, of this type of snake t years after it hatches.

$$\ell(t) = -2^{-(t-2)} + 5$$

Using the graph of $y = 2^t$ below as a starting point, sketch a graph of $y = \ell(t)$, for $0 \le t < 6$, on the axes provided to the right.





c. [5 points] List the transformations you need to apply to the graph of $y = 2^t$ to transform it to that of $y = \ell(t)$. Fill in the first blank with one of the phrases below. Fill in the second blank with a number, "by a factor of" and a number, or N/A for reflections.

Shift it to the left Stretch it horizontally Reflect it across the y-axis

Shift it to the right Compress it horizontally Reflect it across the t-axis

Shift it up Shift it down Stretch it vertically Compress it vertically

First, ______________________________ by __________

then, ______________________________ by __________

then, ______________________________ by __________

then, ______________________________ by __________

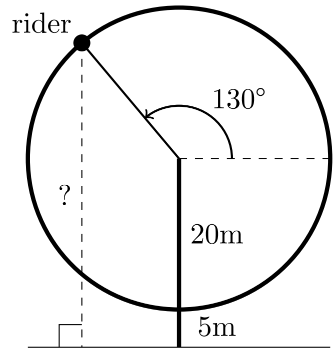
d. [2 points] Give equations for all vertical and horizontal asymptotes of $\ell(t)$. If there are none, write NONE.

Vertical Asymptotes: ____________________

Horizontal Asymptotes: ____________________

6. [10 points] A diagram showing a Ferris wheel is below. The radius of the Ferris wheel is 20 meters, and the lowest point (where people board) is reached by a small set of stairs and is 5 meters above ground level. We'll consider the following question about this scenario:

When an arm of the Ferris wheel is making an angle of $130°$ *with horizontal, how high is that rider off the ground?*



a. [3 points] Ayisha really likes to do trig problems using a unit circle perspective. She comes up with a correct answer that involves $\sin(130°)$ in her answer. What is Ayisha's answer? *Give your answer in exact form.*

________________________ meters

b. [2 points] Oh no! Ayisha accidentally had her calculator in radians when she computed her answer. What would she get in that case? How could she recognize right away that her answer was incorrect?

c. [4 points] Bruno really likes to do trig problems using right triangles. He comes up with a correct answer that involves $\cos(40°)$. Draw the right triangle Bruno could have been considering and use that triangle to find Bruno's correct expression for the rider's height from the ground. *Give your answer in exact form.*

________________________ meters.

d. [1 point] Using a calculator, verify that Ayisha's and Bruno's answers agree. That is, find the numerical value of both expressions. *(We're now assuming that Ayisha's calculator is correctly back in degrees!)*