Math 105 — Final Exam — December 8, 2023

Write your 8-digit UMID number very clearly in the box to the right, and fill out the information on the lines below.

Your Initials Only: _____ Your 8-digit UMID number (not uniqname): _____

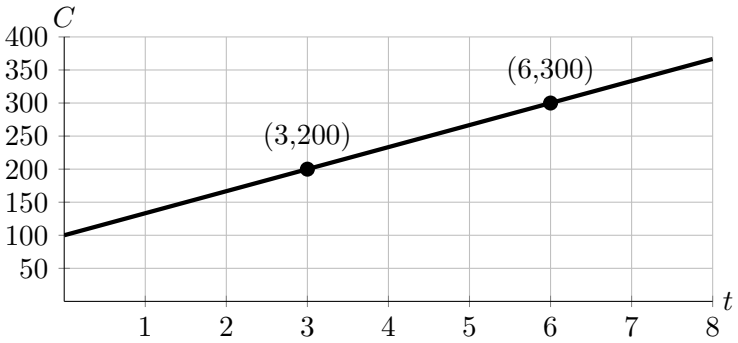
Instructor Name: _____ Section #: _____

1. **Do not open this exam until you are told to do so.**
2. **Do not write your name anywhere on this exam.**
3. Use a pencil for "bubble-in" questions so that you can easily erase your answer if you change your mind.
4. This exam has 12 pages including this cover. There are 9 problems. Note that the problems are not of equal difficulty, so you may want to skip over and return to a problem on which you are stuck.
5. Do not separate the pages of this exam. If they do become separated, write your UMID (not name) on every page and point this out to your instructor when you hand in the exam.
6. The back of every page of the exam is blank, and, if needed, you may use this space for scratch-work. Clearly identify any of this work that you would like to have graded.
7. Read the instructions for each individual problem carefully. One of the skills being tested on this exam is your ability to interpret mathematical questions, so while you may ask for clarification if needed, instructors are generally unable to answer such questions during the exam.
8. Show an appropriate amount of work for each problem, so that graders can see not only your answer but how you obtained it.
9. You must use the methods learned in this course to solve all problems.
10. You are allowed notes written on two sides of a $3'' \times 5''$ note card and one scientific calculator that does not have graphing or internet capabilities.
11. Include units in your answer where that is appropriate.
12. Problems may ask for answers in *exact form* or in *decimal form*. Recall that $\sqrt{2} + \cos(3)$ is in exact form and 0.424 would be the same answer expressed in decimal form.
13. **Turn off all cell phones, smartphones, and other electronic devices**, and remove all headphones, earbuds, and smartwatches. Put all of these items away. The use of any networked device while working on this exam is <u>not</u> permitted.

Problem	Points	Score
1	11	
2	7	
3	8	
4	10	
5	11	

Problem	Points	Score
6	10	
7	7	
8	8	
9	8	
Total	80	

1. [11 points] Pam the Plumber charges customers $P(t)$ dollars for a t hour visit to their house. A graph of $C = P(t)$ is shown below.



a. [2 points] Find the value of $P(0)$ and explain what it means in context.

$P(0) =$ _____

Meaning:

b. [2 points] Find the slope of $P(t)$ and explain what it means in context.

Slope: _____

Meaning:

c. [2 points] Beth the Bathtub-fixer charges \$50 for a home visit plus \$40/hour spent there. Let $B(t)$ be the amount she charges for a t hour visit. Add the graph of $C = B(t)$ for $0 \le t \le 8$ to axes above. **Clearly label at least two points on your graph with their (t, C) coordinates.**

d. [5 points] Find the coordinates of the point where the two graphs intersect and explain what this means in context. *Show all work. Give your final answer in exact form, or rounded to at least two decimal places.*

Point $(t, C) =$ _____

Meaning:

2. [7 points] The amount, in milligrams (mg), of a certain drug in a patient's bloodstream t minutes after it is administered is given by:

$$V(t) = 120e^{-0.006t}$$

a. [2 points] By what percentage does the amount of the drug in the patient's bloodstream decrease each minute? *Show all work. Give your answer in exact form, or rounded to at least* ***three*** *decimal places.*

_____ %

b. [3 points] How long does it take for the amount of the drug in the patient's bloodstream to decrease to 10 mg? *Show all work. Give your answer rounded to the* ***nearest minute****.*

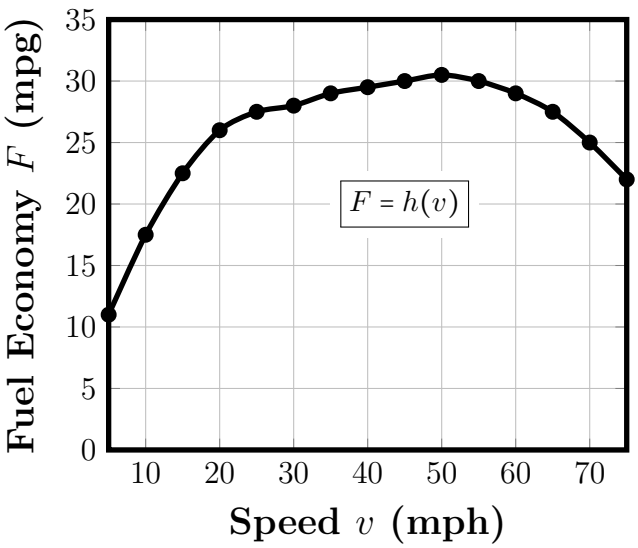
_____ minutes

c. [2 points] The amount, in mg, of a *different* drug in a patient's bloodstream t minutes after it is administered is given by $G(t)$. Some values of $G(t)$ are given below. Could $G(t)$ be exponential? *Show all work.*

t, in minutes	20	30	50
$G(t)$, in mg	95	76	48.64

(Circle One) COULD BE EXPONENTIAL NOT EXPONENTIAL

3. [8 points] The website `fueleconomy.gov` gives the graph on to the right to show fuel economy (in miles per gallon or mpg) as a function of speed (in miles per hour or mph) for a particular type of car. We'll call the function defined by this graph $h(v)$.



a. [1 point] What can you say about the concavity of $h(v)$ over the domain $[10, 30]$? *Circle one answer; no explanation necessary.*

CONCAVE UP

CONCAVE DOWN

NEITHER

b. [1 point] $h(v)$ is not an invertible function. Explain in 1–2 sentences how we know.

Explanation:

c. [1 point] If we wanted to restrict the domain of $h(v)$ so that is *was* an invertible function, what would be a good domain to use?

Domain:_____

Another function $c(F)$, which is invertible, gives the cost of gas in dollars per mile when we have a fuel economy of F mpg.

d. [4 points] Write a sentence or phrase that gives the meaning of each of the following equations or expressions. Or, if it does not make sense in context, explain why not.

i. $c(h(80)) = 0.22$

ii. $c^{-1}(0.18)$

e. [1 point] In this context, what is a reasonable domain for $c(F)$? *No explanation necessary.*

Domain: _____

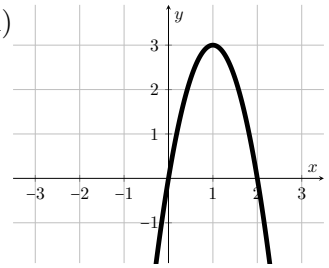
4. [10 points] Below are several quadratic equations, labeled A – J:

A. $y = -3x(x-2)$
B. $y = x(x+2)$
C. $y = x^2 + 1$
D. $y = x^2 - 1$
E. $y = 3x(x+2)$
F. $y = (x-1)(x+1)$
G. $y = -3(x-1)^2 + 3$
H. $y = (x-1)(x+1) + 2$
I. $y = -x(x-2) + 2$
J. $y = (x-1)^2 + 1$

a. [8 points] For each of the graphs below list ALL equations (among A–J) that match the given graph. If no equations match the graph, write NONE.

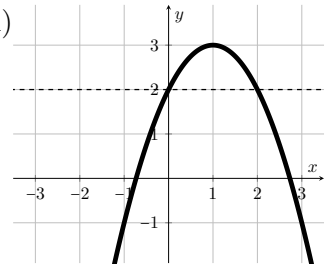
Note that some graphs have dashed lines included, to help you think of those graphs as *shifts* of other quadratic equations.

(i)



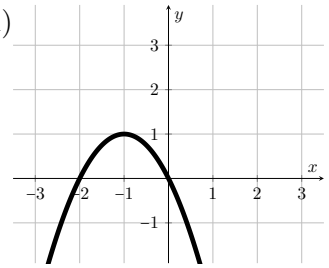
Equation(s): _____

(iii)



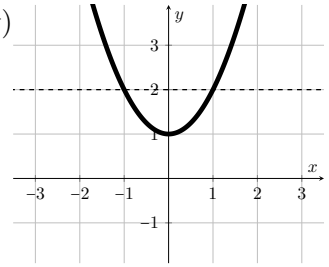
Equation(s): _____

(ii)



Equation(s): _____

(iv)



Equation(s): _____

b. [2 points] It turns out we can also think of these graphs as transformations of each other. Claudia claims that we can transform the graph of (ii) into the graph of (i) using a vertical stretch followed by horizontal shift. She is right! Find the stretch factor and shift value that will complete that transformation:

Stretch vertically by a factor of: _____

Shift right by: _____

5. [11 points] The water levels in a large bay fluctuate due to the tides. High tide, which is when water levels are at their maximum, happens roughly twice per day. Similarly, low tide is when water levels are at their minimum.

a. [6 points] At one particular location in this bay, the depth, in feet, of the water t hours after midnight on December 1 was given by

$$D(t) = 11\cos\left(\frac{24\pi}{149}(t-3)\right) + 56.$$

i. What is the depth of the water, in feet, at this location at high tide? At low tide?

_____ feet at high tide

_____ feet at low tide

ii. Find the period of $D(t)$, either in exact form or rounded to two decimal places. Then interpret what it means in the context of this problem.

Period: _____

Meaning:

iii. Find the times t of all high tides that occur on December 1. *Give your answer as a list of t-values in exact form or rounded to two decimal places.*

$t =$ _____

This problem continues on the next page.

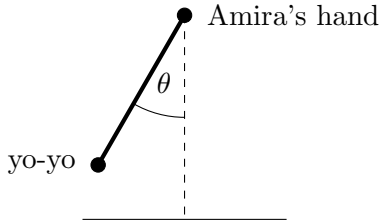
b. [5 points] At another location in the bay, the depth, in feet, of the water t hours after midnight on December 1 was given by

$$P(t) = 9\sin\left(\frac{24\pi}{149}t\right) + 40.$$

Find the t-values of all times on December 1 that the water level at this location was 45 feet. *Give your answer as a list of t-values in exact form or rounded to two decimal places.*

$t =$ _____

6. [10 points] Amira is using a yo-yo as a pendulum by holding the string and letting the yo-yo swing back and forth in a plane—that is, just left to right, not making any kind of ellipse when viewed from above. The symbol θ denotes the maximum angle the string makes with the vertical, as shown in the diagram to the right.



a. [2 points] If θ is $15°$ and the length of the string between Amira's hand and the yo-yo is 3 ft, what is the length of the entire arc that the yo-yo swings through as it travels left to right? *Show all work. Give your answer in exact form or rounded to at least two decimal places.*

Answer: _____ ft

b. [1 point] If Amira adjusted the yo-yo so that the length of the string between her hand the yo-yo were only 1.5 feet instead, how would that change the length of the arc that the yo-yo swings through? *Show your work or explain.*

c. [7 points] Now suppose that

- the length of the string between Amira's hand and the yo-yo is 2 feet,
- at its lowest point, the yo-yo is 1 foot above the ground,
- θ is $\pi/7$ radians,
- and that it takes 1.6 seconds for the yo-yo to make a full swing from left to right and back to left again.

Give all answers below in exact form or rounded to two decimal places.

Find the maximum height of the yo-yo. *Show all work,* ***including a diagram****.*

Answer: _____ ft

Let $h(t)$ be the function giving the height, in feet, of the yo-yo at time t seconds after it is released from its maximum height. Find the amplitude and period of $h(t)$. ***Include units****.*

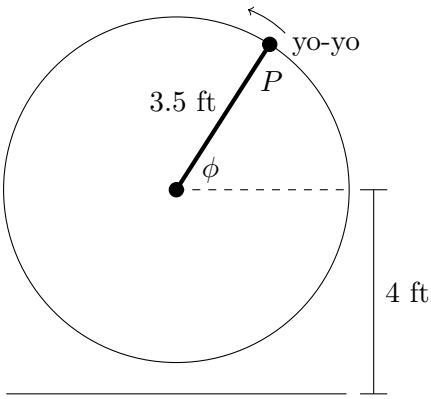
Answer: amplitude = _____

Answer: period = _____

7. [7 points]

Amira's friend Paul borrows her yo-yo and starts spinning it in a counterclockwise circle at a constant speed. His hand holds the string, at the center of the circle shown, 4 feet off the ground. The length of the string between his hand and the yo-yo is 3.5 feet.

In this problem, measure angles counterclockwise from the positive horizontal as usual. When the yo-yo is at point P, the angle ϕ as shown in the diagram to the right is $\frac{3\pi}{8}$ radians.



a. [3 points] How high off the ground is the yo-yo when it is at point P? *Give your answer in exact form or rounded to at least two decimals.*

Height: _____ feet

b. [2 points] After the yo-yo travels most of the way around the circle from its current position, there will be a moment at which it is directly underneath point P. Find the angle, in radians, between 0 and 2π, at which this occurs. *Give your answer in exact form or rounded to at least two decimals.*

Angle: _____ radians

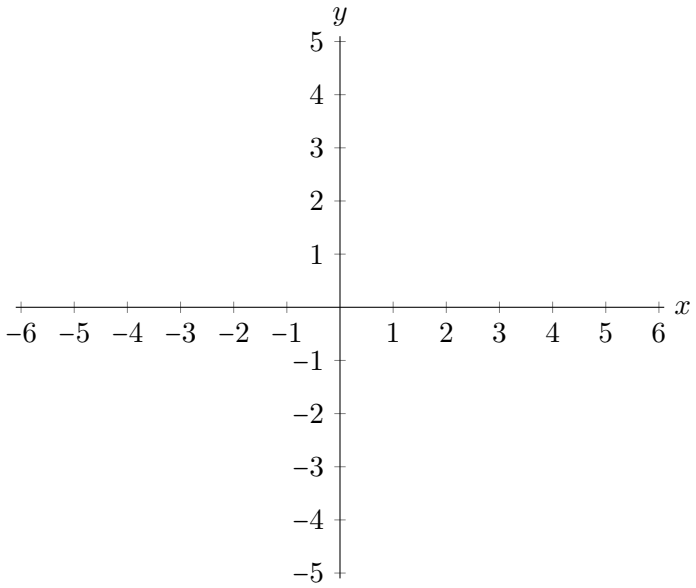
c. [2 points] It takes the yo-yo 1 second to make a complete circle. What angle, in radians, will the yo-yo make with the positive horizontal 1/3 of a second after it is at point P? *Give your answer in exact form or rounded to at least two decimals.*

Angle: _____ radians

8. [8 points]

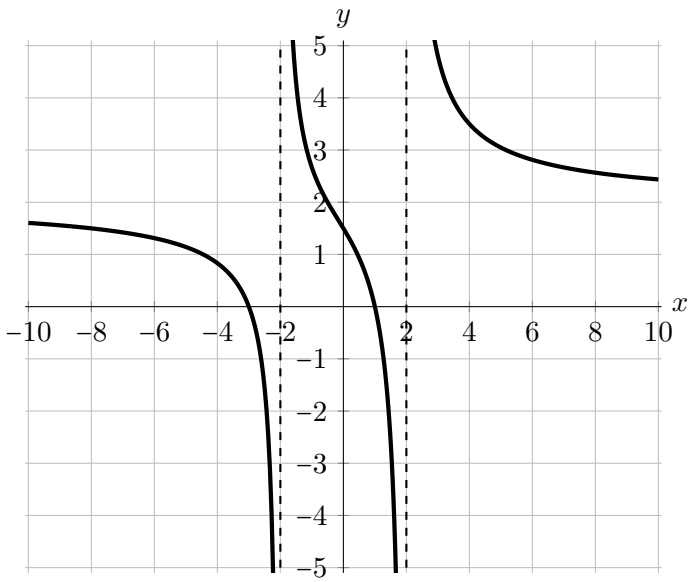
a. [4 points] Sketch a graph of a polynomial $f(x)$ satisfying the following conditions:

- $f(x)$ has zeros at $x = -1$, 2, and 4
- the y-intercept is 1
- $\lim_{x \to -\infty} f(x) = -\infty$
- $f(x)$ is of degree 4



b. [4 points] Write a possible formula for the graph of the rational function shown below. For clarity, its features are also described below.

- the y-intercept is 1.5
- the zeros are -3 and 1.
- horizontal asymptote of $y = 2$
- vertical asymptotes of $x = -2$ and $x = 2$



$y =$ _____

9. [8 points]

a. [5 points] Find the values of the following limits. Your answer may be a numerical value, ∞, or $-\infty$. *You do not need to show work, but limited partial credit may be earned from work shown.*

(i) $\lim_{x\to 2} \dfrac{3(x-1)(x-2)}{(x-2)(x+3)} =$ _____

(ii) $\lim_{x\to \infty} \dfrac{3(x-1)(x-2)}{(x-2)(x+3)} =$ _____

(iii) $\lim_{x\to \infty} \dfrac{x^8 - 7^x}{6^x + x^9} =$ _____

(iv) $\lim_{x\to \infty} \ln(x) =$ _____

b. [3 points] The weight w of a round melon is proportional to the cube of its radius r. That is,

$$w = kr^3,$$

where k is a constant. Currently, the melon's radius is 8 cm, and it weighs 5 pounds. How much would it weigh if its radius were to grow to 12 cm? *Give your answer in exact form or rounded to at least two decimals.*

Answer: _____ pounds