Math 105 — First Midterm — October 8, 2024

Write your 8-digit UMID number very clearly in the box to the right, and fill out the information on the lines below.

Your Initials Only: ______ Your 8-digit UMID number (not uniqname): ________________

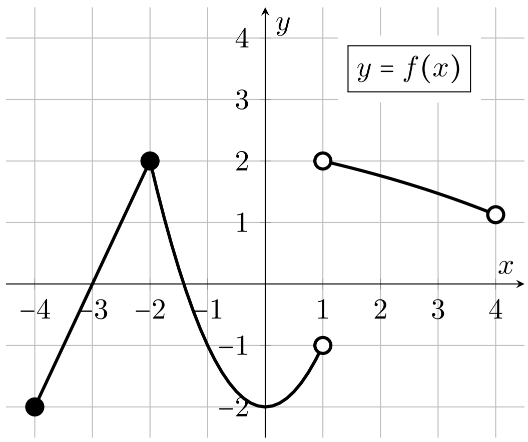
Instructor Name: ________________________________ Section #: __________

1. **Do not open this exam until you are told to do so.**
2. **Do not write your name anywhere on this exam.**
3. This exam has 7 pages including this cover. There are 6 problems. Note that the problems are not of equal difficulty, so you may want to skip over and return to a problem on which you are stuck.
4. Do not separate the pages of this exam. If they do become separated, write your UMID (not name) on every page and point this out to your instructor when you hand in the exam.
5. The back of every page of the exam is blank, and, if needed, you may use this space for scratch-work. Clearly identify any of this work that you would like to have graded.
6. Read the instructions for each individual problem carefully. One of the skills being tested on this exam is your ability to interpret mathematical questions, so while you may ask for clarification if needed, instructors are generally unable to answer such questions during the exam.
7. Show an appropriate amount of work for each problem, so that graders can see not only your answer but how you obtained it.
8. You must use the methods learned in this course to solve all problems.
9. You are allowed notes written on two sides of a $3'' \times 5''$ note card and one scientific calculator that does not have graphing or internet capabilities.
10. Include units in your answer where that is appropriate.
11. Problems may ask for answers in *exact form* or in *decimal form*. Recall that $\sqrt{2} + \cos(3)$ is in exact form and 0.424 would be the same answer expressed in decimal form.
12. **Turn off all cell phones, smartphones, and other electronic devices**, and remove all headphones, earbuds, and smartwatches. Put all of these items away. The use of any networked device while working on this exam is <u>not</u> permitted.

Problem	Points	Score
1	11	
2	11	
3	12	

Problem	Points	Score
4	10	
5	7	
6	9	
Total	60	

1. [11 points] There is information about four different functions below. There is a graph of a function $f(x)$, a piecewise formula for a function $g(x)$, a formula for a function $h(x)$, and a table of some values of an ***invertible*** function $k(x)$.



$$g(x) = \begin{cases} -x & x \leq 0 \\ (x-2)^2 & x > 0 \end{cases}$$

$$h(x) = -2(x-1) + 4$$

x	-2	0	4	6
$k(x)$	4	1	8	12

a. [2 points] Let $m(x) = f(x) + 3$. Find the range of $m(x)$. You can express your answer using either intervals or inequalities.

Range:________________________

b. [2 points] On which of the intervals below is $f(x)$ **decreasing** on the **entire** interval? Circle **all** correct answers.

$(-3,-1)$ $\quad$ $(-2,0)$ $\quad$ $(0,2)$ $\quad$ $(1,3)$ $\quad$ NONE

c. [2 points] Let $b(x)$ be the linear function which is perpendicular to $h(x)$ and which goes through the point $(8,3)$. Find a formula for $b(x)$.

$b(x) =$ ________________________

d. [5 points] Find or estimate the value of each of the following; write N/A if a value does not exist or there is not enough information to find it.

(i) If $w(x) = f(x-3)$, $\quad w(1) =$ ____________

(ii) $k^{-1}(4) =$ ____________

(iii) $k(h(3)) =$ ____________

(iv) The average rate of change of $g(x)$ from $x = -2$ to $x = 3$. ____________

2. [11 points] The population of Detroit in 1970 was 1.51 million and by 1990 it was 1.03 million.

In parts (a)–(c) below, use the following variable definitions:

- Let D be the population of Detroit in **millions of people**.
- Let t be **years since 1970**.

a. [3 points] If we assume the population of Detroit decreased **exponentially** as a function of time, find a formula for $D = E(t)$.

Show all work. Leave constants in exact form, or rounded to at least two decimal places.

$D = E(t) =$ ____________________

b. [2 points] Is the graph of $D = E(t)$ concave up or concave down or neither? Provide a small sketch or explain *briefly* how you know.

(Circle One) CONCAVE UP CONCAVE DOWN

Explanation or Sketch:

c. [3 points] If we assume, instead, that the population of Detroit decreased **linearly** as a function of time, find a formula for $D = L(t)$.

Show all work. Leave constants in exact form, or rounded to at least two decimal places.

$D = L(t) =$ ____________________

d. [3 points] Fort Myers, Florida had a population of 105,260 at the beginning of 2022, which grew by 6.82% over the next year. If the population continues to grow exponentially, how large will the city be in 2030?

Show all work. Leave answer in exact form, or rounded to the nearest whole number.

Population in 2030: ____________________

3. [12 points]

- The amount of money Bizzi earns in one week, W (in US Dollars), is a function of the number of hours she works that week, h. That is, $W = f(h)$.
- On the other hand, the number of hours she can spend watching K-pop videos, K, is *also* a function of the number of hours she works that week, h. That is, $K = g(h)$.

Both $W = f(h)$ and $K = g(h)$ are invertible functions.

a. [6 points] Describe the meaning of each of the following expressions or equations in the context of Bizzi's life, or explain why the expression or equation doesn't make sense in context.

(i) $f^{-1}(1210) = 50$

(ii) $g(f(30)) = 2$

(iii) $g(40)$

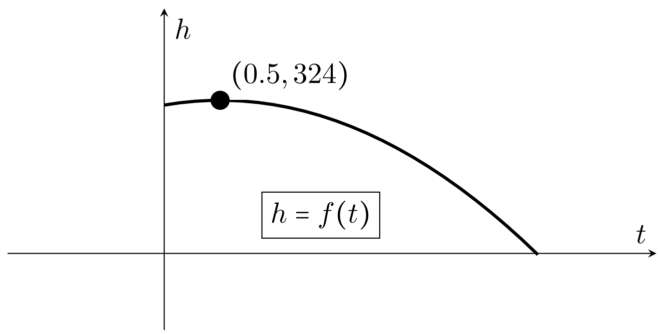
b. [6 points] For each of the following phrases or sentences, write an expression or equation to represent it symbolically using the functions f, g, or their inverses.

(i) Bizzi does *not* work on January 1, but gets paid that week as if she worked 8 hours that day. Express the amount of money Bizzi earns the week of January 1 if she works h hours total during the remainder of the week.

(ii) The week of April 1, every employee in the company gets a \$200 bonus. Express the amount of money Bizzi earns the week of April 1 if she works h hours.

(iii) Represent the following sentence as an equation: On a week where Bizzi earns 950 dollars, she has time to watch 3 hours of K-Pop videos.

4. [10 points] The AT&T Building in Detroit (photo below, left) is 320 feet tall. A ball is (safely) launched from the level of the roof of the AT&T Building. Its height above the ground h, in feet, is a quadratic function of time t, in seconds. A graph of $h = f(t)$ is shown below, to the right. The parabola's vertex is the point $(0.5, 324)$.



a. [2 points] Given the context of this problem, what is the vertical intercept of the graph $h = f(t)$? Explain how you know.

Vertical intercept: ____________________

Explanation:

b. [2 points] How many seconds after launch did the ball reach its maximum height and how high was that?

Seconds after launch when maximum height was reached: _____________

Maximum height reached: ____________________ feet

c. [3 points] Use the information from parts (a) and (b) to find a formula for $h = f(t)$. *Show all work.*

$f(t) =$ ___________________________

This problem continues on the next page.

d. [3 points] A ball is thrown down from a hovercraft cruising above *Jupiter*. The ball's height above Jupiter's surface, h, in feet, is given by:

$$h = -40t^2 - 20t + 560$$

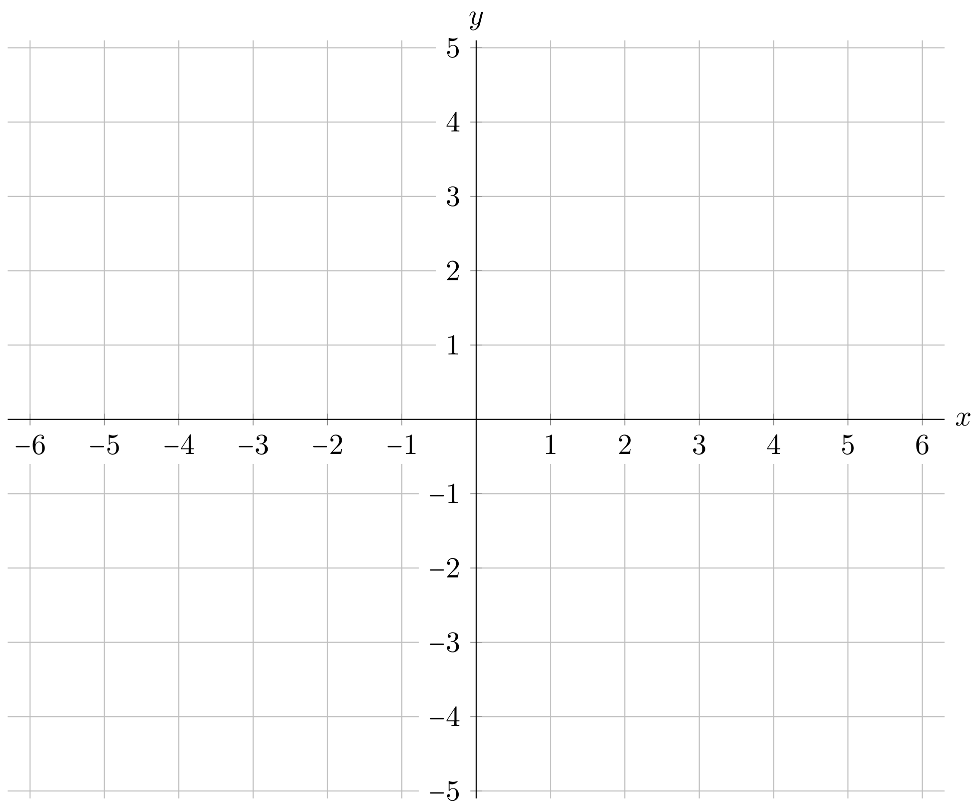
where t is measured in seconds after the ball was released.

From the moment the ball was released, how many seconds did it take for the ball to reach the surface of Jupiter? *Show all work. Give your answer in exact form, or rounded to at least two decimal places.*

________________________seconds

5. [7 points] On the axes below, sketch a graph of a **single function** $j(x)$ that satisfies all of the following properties:

- $j(x)$ has zeros at $x = 1$ and $x = 3$.
- The domain of $j(x)$ is $-6 \le x < \infty$.
- $j(x) \longrightarrow -2$ as $x \longrightarrow \infty$. In other notation: $\lim_{x\to\infty} j(x) = -2$.
- $j(x)$ is decreasing on the interval $[-6, -4]$.
- The average rate of change of $j(x)$ on the interval $-3 \le x \le -1$ is 2.
- $j(x)$ is concave down on the interval $1 < x < 3$.



6. [9 points] Isabel's friend Kei lives in the next town over. The two friends are curious about how their water bills compare. Let $I(w)$ be the amount, in dollars, Isabel pays for her water bill for a month if she uses w Centum Cubic Feet (CCFs) of water that month. Let $K(w)$ be the amount, in dollars, Kei pays for their water bill for a month if they use w CCFs of water that month. Both functions are linear and their formulas are:

$$I(w) = 4.1w + 25 \qquad K(w) = 4.5w + 15$$

a. [3 points] Find $K^{-1}(33)$ and write a sentence which explains what the value you find means in the context of the problem. *Show all work. Give your answer in exact form or rounded to at least two decimal places.*

$K^{-1}(33) =$ ____________________

Meaning:

b. [1 point] If Kei used two more CCFs of water in August than in June, how much more expensive was their August water bill than their June water bill? *You do not need to show any work.*

Kei's August water bill is ______________dollars more than their June water bill.

c. [2 points] What is the amount of water usage (in CCFs) that would cost the same amount under both water bill plans? *Show all work. Give your answer in exact form, or rounded to at least two decimal places.*

______________CCFs

Let $g(t)$ be the number of CCFs of water Kei's household has used t days since the start of June (so $t = 1$ would correspond to 12:00am on June 2nd). Some values of $g(t)$ are displayed in the table below.

t	1	5	7	11
$g(t)$	5	19.5	28	33

d. [3 points] Kei's family went out of town (and therefore didn't use any water at home) for a couple days during June. Based on the table above, during which of the following time periods is most likely that Kei's family went out of town?

Circle the **one** best possible answer. *Show all work and explain why you circled the option you chose.*

June 3rd to June 5th June 6th to June 8th June 9th to June 11th

Explanation: