Math 105 — Final Exam — December 12, 2024

Write your 8-digit UMID number very clearly in the box to the right, and fill out the information on the lines below.	

Your Initials Only:	Your 8-digit UMID number (not uniquame):
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Instructor Name:	Section #:

1. Do not open this exam until you are told to do so.

2. Do not write your name anywhere on this exam.

- 3. This exam has 11 pages including this cover. There are 9 problems. Note that the problems are not of equal difficulty, so you may want to skip over and return to a problem on which you are stuck.
- 4. Do not separate the pages of this exam. If they do become separated, write your UMID (not name) on every page and point this out to your instructor when you hand in the exam.
- 5. The back of every page of the exam is blank, and, if needed, you may use this space for scratch-work. Clearly identify any of this work that you would like to have graded.
- 6. Read the instructions for each individual problem carefully. One of the skills being tested on this exam is your ability to interpret mathematical questions, so while you may ask for clarification if needed, instructors are generally unable to answer such questions during the exam.
- 7. Show an appropriate amount of work for each problem, so that graders can see not only your answer but how you obtained it.
- 8. You must use the methods learned in this course to solve all problems.
- 9. You are allowed notes written on two sides of a $3'' \times 5''$ note card and one scientific calculator that does not have graphing or internet capabilities.
- 10. Include units in your answer where that is appropriate.
- 11. Problems may ask for answers in *exact form* or in *decimal form*. Recall that $\sqrt{2} + \cos(3)$ is in exact form and 0.424 would be the same answer expressed in decimal form.
- 12. Turn off all cell phones, smartphones, and other electronic devices, and remove all headphones, earbuds, and smartwatches. Put all of these items away. The use of any networked device while working on this exam is <u>not</u> permitted.

Problem	Points	Score
1	14	
2	14	
3	6	
4	6	
5	6	

Problem	Points	Score
6	9	
7	7	
8	11	
9	7	
Total	80	

1. [14 points]

The entire graph of a function f(x) is shown to the right. Also shown is a table of some values for two functions g(x) and h(x).

x	0	1	2	4
g(x)	2	-1	5	4
h(x)	0	0.5	0	8

The function g(x) is defined for all real numbers and is **periodic** with a period of 5.



- a. [3 points] Find the value of each of the following; write N/A if a value does not exist or there is not enough information to find it. Showing work is not required, but may be eligible for partial credit in some cases.
 - (i) h(f(1)) = ______
 - (ii) $g(\sin(20\pi)) =$ _____
 - (iii) $g(f^{-1}(-1)) =$ _____
- b. [3 points] Suppose that we know further that h(x) is a polynomial of **degree 3** with a double zero at x = 2. Combining this new knowledge with what's given in the table, find a formula for h(x). Show all work.

h(*x*) = _____

c. [6 points] The piecewise function f(x) consists of a linear piece and an exponential piece. Write a piecewise formula for the function f(x). Show all needed work.



d. [2 points] Find the domain of the function $f^{-1}(y)$ (not f(x)).

Domain of $f^{-1}(y)$: _____

- 2. [14 points] Psychiatrists and food scientists teamed up to measure how the concentration of different sweeteners affect the perception of sweetness. "Sweetness units" (SU) in this trial ranged from 1 to 15.
 - The function F(C) gives the sweetness units (SU) of a C% fructose solution. (A concentration of C% means that, by mass, the fructose was C% of the total solution and the rest was water.)
 - The function A(C) gives the sweetness units (SU) of a C% Alitame solution. (A concentration of C% means that, by mass, the Alitame was C% of the total solution and the rest was water.)
 - **a**. [6 points] Write an expression or equation, using A, A^{-1}, F, F^{-1} , or their combinations or compositions, that represents each of the following sentences or phrases.
 - (i) The perceived sweetness of a 6% fructose solution is 9 SUs.
 - (ii) The concentration of Alitame that gives a perceived sweetness of 8 SU
 - (iii) A 0.01% solution of Alitame is 100 times sweeter than a 1% solution of fructose.

- **b.** [2 points] Which of the following compositions make sense in the context of the problem? *Bubble in all that apply.*
 - $\bigcirc A(F(15)) \\ \bigcirc F(A(0.05)) \\ \bigcirc A^{-1}(F(11)) \\ \bigcirc A(F^{-1}(10)) \\ \bigcirc A^{-1}(F^{-1}(5)) \\ \bigcirc \text{ NONE OF THE ABOVE}$

This information is repeated from the previous page for convenience.

- The function F(C) gives the sweetness units (SU) of a C% fructose solution. (A concentration of C% means that, by mass, the Fructose was C% of the total solution and the rest was water.)
- The function A(C) gives the sweetness units (SU) of a C% Alitame solution. (A concentration of C% means that, by mass, the Alitame was C% of the total solution and the rest was water.)
- c. [3 points] The scientists found that F(C) was linear, with a slope of 1.33. Given that, which of the following statements about F(C) are TRUE? Bubble in all that apply.

 \bigcirc A 10% fructose solution will be 33% sweeter than a 9% fructose solution.

 \bigcirc An 8% fructose solution will be about 4 SUs sweeter than a 5% fructose solution.

 \bigcirc A 4% fructose solution will be 1.33 SUs *less sweet* than a 5% fructose solution.

- \bigcap F(C) is an increasing function.
- $\bigcap F(C)$ has a constant average rate of change.
- NONE OF THE ABOVE
- d. [3 points] The scientists published the following table of data for the function A(C).

C	0.0025	0.005	0.01	0.02
A(C)	6.89	9.36	11.4	12.8

Which of the following statements about A(C) may be TRUE, given data in the table provided. Bubble in all that apply.

- () A(C) is increasing.
- \bigcirc The average rates of change of A(C) are decreasing.
- $\bigcirc A(C)$ is linear.
- () A(C) is concave up.
- () A(C) is concave down.
- () NONE OF THE ABOVE

3. [6 points] For each of the following equations, bubble in the letter of the corresponding graph or bubble in **D** if it does not correspond to any of the shown graphs. Use pencil in case you need to change your answer. A graph may appear as an answer multiple times.



4. [6 points] The entire graph of f(x) is show below. For each subsequent graph show below, write a formula for the graph shown in terms of f(x). That is, write the formula as a transformation of f(x). Or if it cannot be written as a transformation of f(x), write N/A.



- 5. [6 points] The parts of the question below are unrelated.
 - **a**. [3 points] The circle below is centered at the origin and has a radius of 2.5 meters. The arc shown in bold has a length of 5 meters. Give the coordinates of P in exact form or rounded to at least three decimal places.



Coordinates of P =_____

b. [3 points] Trying to find the cosine of 310 *degrees*, Sofia enters cos(310) into her calculator and gets -0.52534763851. How does she immediately know that something is wrong? *Draw a picture* and explain in words how Sofia knows her calculator must not have been in "degree mode"?

- 6. [9 points] Bubble in the blanks for all possible correct choices. Use pencil in case you need to change your answer. You do not need to show work for any part of this problem.
 - **a**. Which of the functions below have the property that: $\lim_{x \to \infty} f(x) = \infty$?
 - $f(x) = \frac{9}{x^5} \qquad \qquad f(x) = \frac{x^{\frac{1}{3}}}{\ln(x)}$ $f(x) = \frac{x^9 + 5}{2^x + x^2} \qquad \qquad O \text{ NONE OF THE ABOVE}$ $f(x) = \frac{x^2}{e^{-x}}$

b. Which of the following functions have at least one *horizontal* asymptote?

$$f(x) = \log(5x)$$

$$f(x) = e^{3x} + 1$$

$$f(x) = \frac{5x^5 - x^2}{x^5 + x^4}$$

$$f(x) = \frac{x^3 + x + 71}{x^2 - 81}$$

$$f(x) = \frac{2}{x^3 - x - 17}$$

$$NONE OF THE ABOVE$$

c. Which of the following functions have at least one *vertical* asymptote?

$$\int f(x) = \frac{1}{x-5} \qquad \qquad \bigcirc f(x) = \frac{x^2 - 2x + 5}{x^2 + 1}$$

$$\int f(x) = \ln(x) + 5 \qquad \qquad \bigcirc \text{ NONE OF THE ABOVE}$$

$$\int f(x) = \frac{x^2(x-1)^2}{(x-1)}$$

d. In which of the following equations is y directly proportional to x^2 ?

$$\bigcirc y = 2x$$

$$\bigcirc y = 2x^{2}$$

$$\bigcirc y = \frac{4}{x^{2}}$$

$$\bigcirc y = \frac{4}{x^{2}}$$

$$\bigcirc y = \frac{4}{x^{2}}$$

$$\bigcirc y = \frac{\sqrt{7x^{2}}}{3}$$

$$\bigcirc \text{ NONE OF THE ABOVE}$$

- 7. [7 points] On the axes below, sketch a graph of a single function y = g(x) that satisfies all of the following properties:
 - the domain of g(x) is all real numbers except for -2 and 3
 - the average rate of change of g(x) on the interval $-5 \le x \le -3$ is 3/2
 - g(x) has a vertical asymptote at x = -2
 - g(x) has a hole at x = 3
 - g(x) is concave up on the interval $(4, \infty)$
 - g(x) has a horizontal asymptote at y = -3



8. [11 points] In US households, electrical voltage (in volts) can be modeled by the function

$$V(t) = 155.6 \sin(120\pi t)$$

where t is measured in seconds.

a. [4 points] On the axes below, sketch a graph of **two periods** of y = V(t). Your second cycle should end at exactly the furthest right tick on the *t*-axis. Clearly label at least two ticks on the *t*-axis. Use the ticks on the *y*-axis for your maximum and minimum values of V(t) and label them as well.



b. [4 points] Find the first three positive values of t where the voltage is equal to 120 volts. Show all work. Leave your answers in exact form or round to at least four decimal places.

t = _____, _____, _____, _____,

c. [3 points] In Australia, the voltage alternates between a maximum of 240 volts, to a minimum of -240 volts, and back to 240 volts 50 times per second. Find a formula for the function A(t) which models the voltage in Australia t seconds from when the voltage is at its maximum.

9. [7 points] Jay drinks a cup of coffee which contains 140mg of caffeine. The amount of caffeine, in milligrams, left in his body h hours after he drinks the coffee is given by the function

$$C(h) = 140(0.89)^{h}$$

For each part below, <u>show all work</u> and leave your answer in exact form or rounded to at least two decimal places.

a. [2 points] What percent of the caffeine in the cup of coffee is left in Jay's body 2 hours after he drinks it?

b. [2 points] What is the continuous percent decay rate of C(h)?

c. [3 points] Find the half-life of C(h). Include relevant units in your answer.

_%.

_%