Math 105 — Second Midterm

March 17, 2016

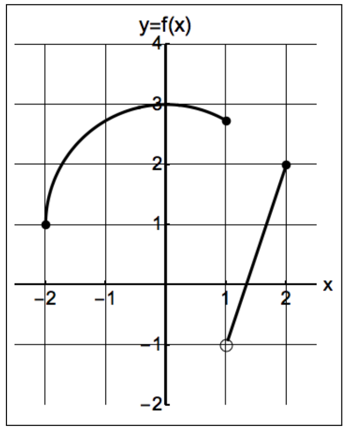
Uniquename: EXAM SOLUTIONS Initials: ________

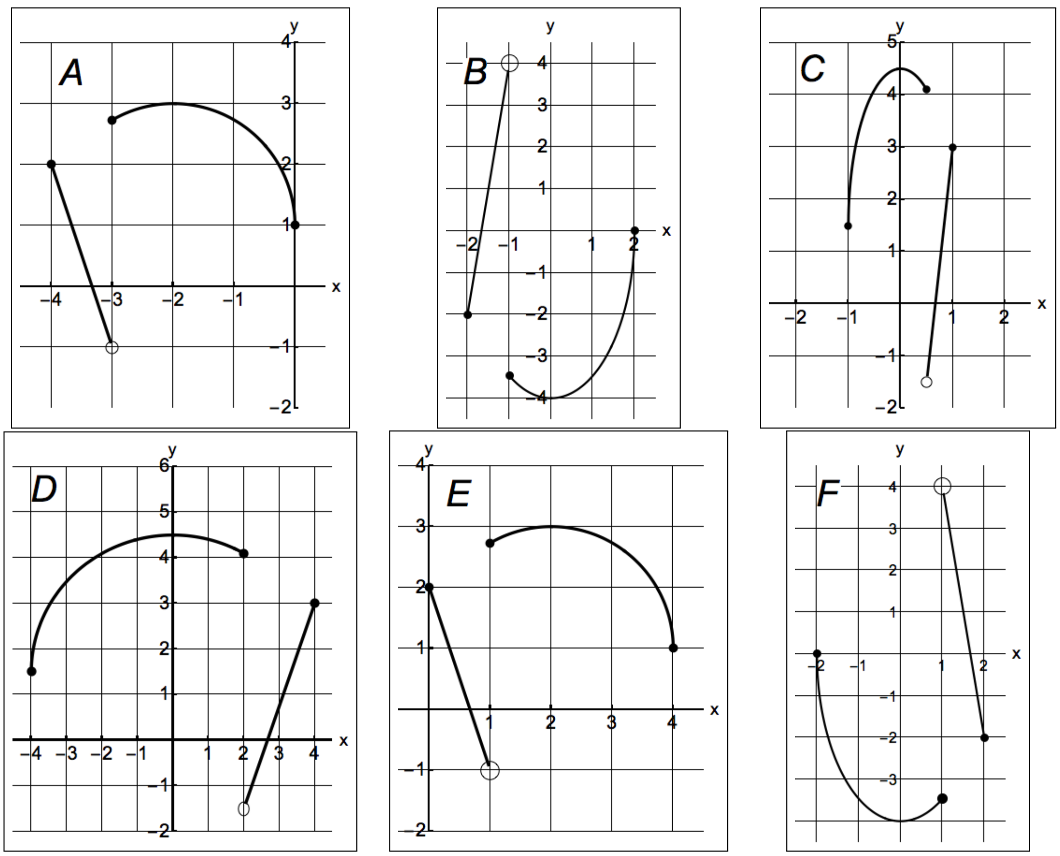
Instructor: ________ Section: ________

1. **Do not open this exam until you are told to do so.**
2. This exam has 10 pages including this cover. There are 9 problems. Note that the problems are not of equal difficulty, so you may want to skip over and return to a problem on which you are stuck.
3. Do not separate the pages of this exam. If they do become separated, write your name on every page and point this out to your instructor when you hand in the exam.
4. Please read the instructions for each individual problem carefully. One of the skills being tested on this exam is your ability to interpret mathematical questions, so instructors will not answer questions about exam problems during the exam.
5. Show an appropriate amount of work (including appropriate explanation) for each problem, so that graders can see not only your answer but how you obtained it. Include units in your answer where that is appropriate.
6. You may use any calculator except a TI-92 (or other calculator with a full alphanumeric keypad). However, you must show work for any calculation which we have learned how to do in this course. Notecards are not allowed in this exam.
7. If you use graphs or tables to find an answer, be sure to include an explanation and sketch of the graph, and to write out the entries of the table that you use.
8. **Turn off all cell phones and pagers**, and remove all headphones.
9. You must use the methods learned in this course to solve all problems.

Problem	Points	Score
1	8	
2	16	
3	12	
4	10	
5	9	
6	12	
7	10	
8	11	
9	12	
Total	100	

1. [8 points] The graph of the function $y = f(x)$ is shown below. Write the letter (A-F) of the graph that corresponds to each of the functions listed below. If none of the graphs corresponds to the function write“NONE”.





Solution:

$y = 1.5f(2x)$: **C**

$y = f(2 - x)$: **E**

$y = 1 - 2f(x)$: **None**

$y = 2(1 - f(x))$: **F**

2. [16 points]

a. [4 points] The domain and range of the function $y = f(x)$ are $[-2, 6)$ and $(-\infty, -10]$, respectively. What is the domain and range of $g(x) = 1 - f(\frac{1}{4}(x + 8))$?

Solution: Domain: $[-16, 16)$ Range: $[11, \infty)$

b. [2 points] If $f(x) = |x^3|$, then the function $f(x)$ is (circle your answer)

Solution: $\boxed{\text{EVEN}}$ ODD NEITHER

c. [2 points] Complete the following sentence:

Solution: If $f(x) = 2^x$, then the graph of $g(x) = f(x + 3)$ can be obtained by applying a vertical stretch by a factor of **8** to the graph of $y = f(x)$.

d. [4 points] Find the equations of the vertical and horizontal asymptotes (if any) of the following functions. If a function does not have vertical or horizontal asymptotes write "None".

Solution:

i) $y = 3e^{-0.4x} - 2$

Vertical asymptote: **None** Horizontal asymptote: $y = -2$.

ii) $y = 1 - 7\log(3x + 1)$

Vertical asymptote: $x = -\frac{1}{3}$ Horizontal asymptote: **None**

e. [2 points] Find two exact values of $-\pi < \theta \leq \pi$, measured in radians, such that $\cos\theta = \cos(A)$, where $A = \frac{11}{5}\pi$ radians.

Solution:

$$\theta = \frac{1}{5}\pi, -\frac{1}{5}\pi.$$

f. [2 points] Let $f(x)$ be a periodic function that has amplitude 4 and let $g(x) = 3f(5x)$. Find the amplitude of the function $g(x)$.

Solution: Amplitude of $g(x) = 12$

3. [12 points] Let $C(t)$ and $A(t)$ be the production (in thousands of pounds) of corn and apples in a farm t years after 2002, where

$$C(t) = 200e^{-0.4t+1} \qquad \text{and} \qquad A(t) = 120e^{0.5t}$$

a. [3 points] What is the annual percent growth rate of the function $C(t)$? Your answer must be exact or accurate up to the first two decimals. Show all your work.

Solution: Since $C(t) = 200e^{-0.4t+1}$, then $b = e^{-0.4}$ and $r = b - 1 = e^{-0.4} - 1 \approx -0.32$.

b. [4 points] How long after 2002 will the production of corn be reduced to a third of its size in that year? Your answer must be exact or accurate up to the first two decimals. Show all your work.

Solution:

$$\begin{aligned} 200e^{-0.4t+1} &= \frac{200}{3}e \\ e^{-0.4t} &= \frac{1}{3} \\ -0.4t &= \ln\left(\frac{1}{3}\right) \qquad t = -\frac{1}{0.4}\ln\left(\frac{1}{3}\right) \approx 2.74 \text{ years after 2002.} \end{aligned}$$

c. [5 points] According to these functions, when will the production of corn be the same as the production of apples? Your answer must be in **exact form**. Show all your work.

Solution:

$$\begin{aligned} 200e^{-0.4t+1} &= 120e^{0.5t} \\ 200e(e^{-0.4t}) &= 120e^{0.5t} \\ \frac{200e}{120} &= \frac{e^{0.5t}}{e^{-0.4t}} \\ e^{0.9t} &= \frac{5}{3}e \\ 0.9t &= \ln\left(\frac{5}{3}e\right) \\ t &= \frac{1}{0.9}\ln\left(\frac{5}{3}e\right). \end{aligned}$$

4. [10 points] A new drug, Lexicor, helps reduce the symptoms of the common cold. Doctors recommend to take Lexicor the moment a patient starts showing symptoms of a cold. Let

$$T(x) = 200 - 150\log(ax+3)$$

be the length of time (in hours) needed for the drug to eliminate the common cold symptoms after a dose of x mg. In this problem a is a nonzero constant.

a. [2 points] According to the function $T(x)$, how long will it take for the symptoms of the common cold to disappear, after a patient starts showing symptoms of a cold, if he does not take Lexicor? *Include units.*

Solution:

$$T(0) = 200 - 150\log(a(0)+3) = 200 - 150\log(3) \approx 128.43 \text{ hours.}$$

b. [4 points] List all the transformations, in order, that you need to apply to the graph of the function $f(x) = 150\log(x)$ in order to get the graph of the function $y = T(x)$. Assume that $0 < a < 1$. Make sure to write each transformation carefully.

Solution:

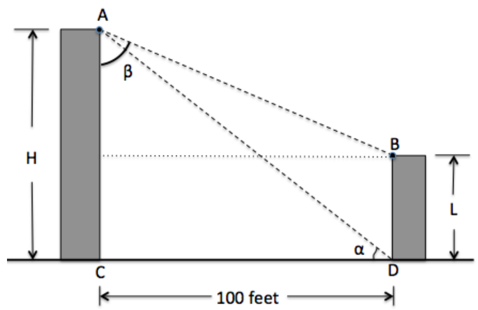
1)Horizontal shift to the left by 3.
2)Horizontal stretch by $\frac{1}{a}$.
3)Reflection about the x-axis.
4)Vertical shift up by 200.

c. [4 points] Find the value of the constant a if the symptoms of the common cold are eliminated 25 hours after taking a dose of 300 mg of Lexicor. Your answer must be exact or include at least three decimals. Show all your work.

Solution:

$$\begin{aligned} 200 - 150\log(300a+3) &= 25 \\ -150\log(300a+3) &= -175 \\ \log(300a+3) &= \frac{175}{150} = \frac{7}{6} \\ 300a+3 &= 10^{\frac{7}{6}} \\ 300a &= 10^{\frac{7}{6}} - 3 \\ a &= \frac{1}{300}\left(10^{\frac{7}{6}} - 3\right) \approx 0.0389. \end{aligned}$$

5. [9 points] Jimmy is at the top of a building at point A (see the diagram below). He is trying to determine the heights H and L of the building at which he is standing and another building that is 100 feet away. He finds out that the angles $\alpha = ADC$ and $\beta = BAC$ measure $37°$ and $65°$ respectively.



a. [2 points] Find a formula for the length of the segment AD in terms of the height H of the building at which Jimmy is standing.

Solution: $AD = \sqrt{100^2 + H^2}$.

b. [3 points] Find the height H of the building in which Jimmy is standing. Include units. Your answer must be exact or include at least two decimals. Show all your work.

Solution: $\tan\alpha = \dfrac{H}{100}$, then $H = 100\tan(37^o) \approx 75.35$ feet.

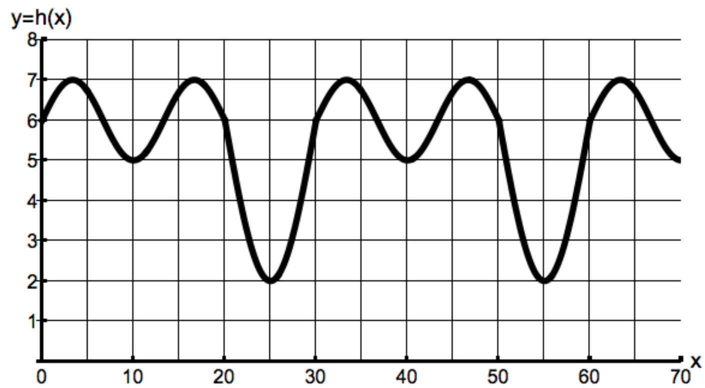
c. [4 points] Find the height L of the building that is 100 feet away. Include units. Your answer must be exact or include at least two decimals. Show all your work.

Solution: Since $\tan\beta = \dfrac{100}{H-L}$, then $H - L = \dfrac{100}{\tan\beta}$.

Hence $L = H - \dfrac{100}{\tan\beta} \approx 28.72$ feet.

6. [12 points]

a. [6 points] Scientists have been recording the number of cases of an infectious disease. They have found that the number of cases reported changes periodically over time, with a period less than 70 weeks. Let $h(x)$ be the average number of cases (in thousands) reported x weeks after the first week of January 2014. The graph of $y = h(x)$ is shown below.



Find the period, amplitude and the equation of the midline of the function $y = h(x)$.

Solution: Period$= 30$ Amplitude$= 2.5$ Midline: $y = 4.5$

b. [3 points] Let $f(x)$ be a periodic function, with period equal to 7, whose domain is all the real numbers. Some of the values of the function $f(x)$ are shown below.

x	-4	-2	0	2
$f(x)$	1	4	7	10

Find the value of the following values of $f(x)$. Write "NP" if it is not possible to determine the value of the function with the information given to you.

Solution:
$f(3) = 1$ $f(8) =$NP $f(-5) = 10$

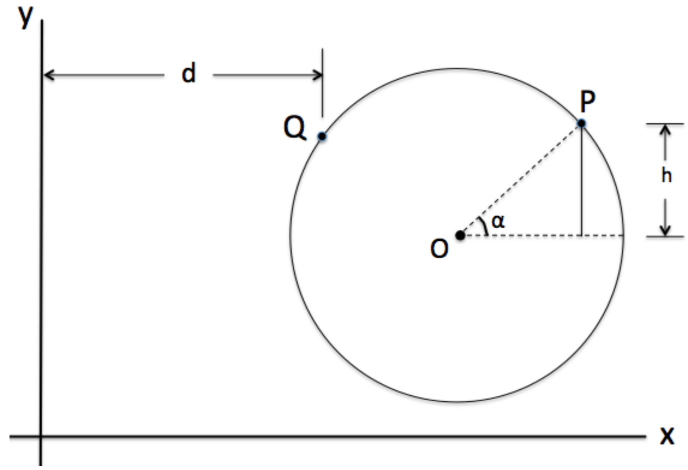
c. [3 points] Some of the values of an odd function $g(x)$ are shown below

x	-6	-1	3	4
$g(x)$	1	-5	7	-10

Find the value of the following values of $g(x)$ assuming that the function is defined for all real numbers. Write "NP" if it is not possible to determine the value of the function with the information given to you.

Solution:
$g(-3) = -7$ $g(0) = 0$ $g(5) =$NP

7. [10 points] Let C be a circle lying entirely in the first quadrant with radius 4 meters and center at the point $O = (a, b)$ (see the diagram below). A spider is standing at the point P on the circle. The point P makes an angle $\alpha = \frac{\pi}{4}$ radians (measured counterclockwise) with the horizontal line passing through the point O.



a. [2 points] Find the length of the vertical distance h from the point P to the horizontal line passing through the center O of the circle.

Solution: $h = 4\sin\left(\frac{\pi}{4}\right) = 4\left(\frac{1}{\sqrt{2}}\right) = \frac{4}{\sqrt{2}} \approx 2.828.$

b. [3 points] The spider walks 7 meters around the circle, in the counterclockwise direction, from the point P until it reaches the point Q. Find the measure of the angle POQ (in radians).

Solution: Using the arclength formula $s = r\theta$ with $\theta =$ angle POQ, we have $\theta =$ angle $POQ = \frac{7}{4}$ radians.

c. [5 points] Find the horizontal distance d, in meters, between the point Q and the y-axis. Your answer must be in **exact form** and may contain the constants a and/or b.

Solution: $d = a + 4\cos\left(\frac{\pi}{4} + \frac{7}{4}\right).$

8. [11 points]

a. [7 points] In 2000, Jesse deposited 5 thousand dollars in a bank account with a continuous interest rate of 13% per year. Ten years later, she deposited 7 thousand dollars in the same account. Suppose that Jesse does not withdraw or deposit any more money into the account.

i) How much money was in the bank account, right before Jesse deposited the 7 thousand dollars? Your answer must be exact or accurate up to the closest cent. Show all your work.

Solution: He will have $5000e^{0.13(10)} \approx 18,346.48$ dollars.

ii) What is the balance, in dollars, in the bank account 16 years after the initial deposit? Your answer must be exact or accurate up to the closest cent. Show all your work.

Solution: He will have $(5000e^{0.13(10)} + 7000)e^{0.13(6)} \approx 55,292.65$ dollars.

b. [4 points] Solve for x in the following equation. Your answer must be in **exact form**. Show all your work carefully to receive full credit.

$$\ln\left(\frac{1}{2}e^{-0.3x} + 1\right) = 4.$$

Solution:

$$\begin{aligned}
\ln\left(\frac{1}{2}e^{-0.3x} + 1\right) &= 4 \\
\frac{1}{2}e^{-0.3x} + 1 &= e^4 \\
e^{-0.3x} &= 2(e^4 - 1) \\
-0.3x &= \ln(2(e^4 - 1)) \\
x &= -\frac{1}{0.3}\ln(2(e^4 - 1))
\end{aligned}$$

9. [12 points] Patrick has an aquarium that has fish of different colors. He has noticed that the lengths of each type of fish are related. Let $B(z)$ be the length, in centimeters, of a blue fish that is z months old.

a. [4 points] The length of a pink fish is always 25 percent shorter than the length of a blue fish of its same age. Let $P(w)$ be the length, in centimeters, of a pink fish that is w **years** old. Find a formula for $P(w)$ in terms of the function B.

Solution:

$$P(w) = 0.75B(12w)$$

b. [4 points] The length of a green fish is equal to the length of a blue fish that is 4 months older. Let $G(y)$ be the length of a green fish, **in millimeters**, that is y months old. Find a formula for $G(y)$ in terms of the function B. *Note: 1 centimeter=10 millimeters.*

Solution:

$$G(y) = 10B(y + 4)$$

c. [4 points] Patrick took some measurements of the lengths, in centimeters, of a blue and a black fish. Consider the following tables of values of the functions $B(z)$ and $F(z)$, where $F(z)$ is the length, in centimeters, of a black fish that is z months old.

z	2	4	6	8
$B(z)$	5	7	10	15

z	4	8	12	16
$F(z)$	2	4	7	12

Find a formula for $F(z)$ in terms of the function B.

Solution:

$$F(z) = B\left(\frac{z}{2}\right) - 3$$