Math 105 — Second Midterm

March 16, 2017

UMID: EXAM SOLUTIONS Initials: ____________

Instructor: ____________ Section: ____________

1. **Do not open this exam until you are told to do so.**
2. **Do not write your name anywhere on this exam.**
3. This exam has 9 pages including this cover. There are 9 problems. Note that the problems are not of equal difficulty, so you may want to skip over and return to a problem on which you are stuck.
4. Do not separate the pages of this exam. If they do become separated, write your UMID on every page and point this out to your instructor when you hand in the exam.
5. Please read the instructions for each individual problem carefully. One of the skills being tested on this exam is your ability to interpret mathematical questions, so instructors will not answer questions about exam problems during the exam.
6. Show an appropriate amount of work (including appropriate explanation) for each problem so that graders can see not only your answer, but also how you obtained it. Include units in your answer where that is appropriate.
7. You may use a TI-84, TI-89, TI-Nspire or other approved calculator. However, you must show work for any calculation which we have learned how to do in this course.
8. If you use graphs or tables to find an answer, be sure to include an explanation and sketch of the graph, and to write out the entries of the table that you use.
9. **Turn off all cell phones, pagers, and smartwatches**, and remove all headphones.

Problem	Points	Score
1	15	
2	11	
3	17	
4	11	
5	8	
6	15	
7	8	
8	7	
9	8	
Total	100	

1. [15 points]

a. [5 points] Suppose $f(x)$ is a function with domain $[-2, 5]$ and range $[7, 12]$. What are the domain and range of the transformation $g(x) = -f(2x + 1) + 2$?

The domain of $g(x)$ is $\underline{\quad [-1.5, 2] \quad}$.

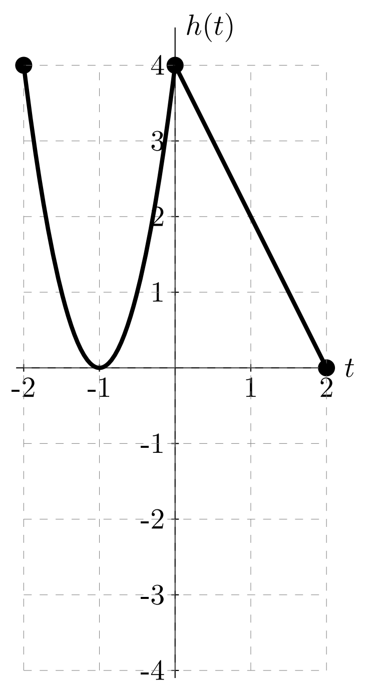
The range of $g(x)$ is $\underline{\quad [-10, -5] \quad}$.

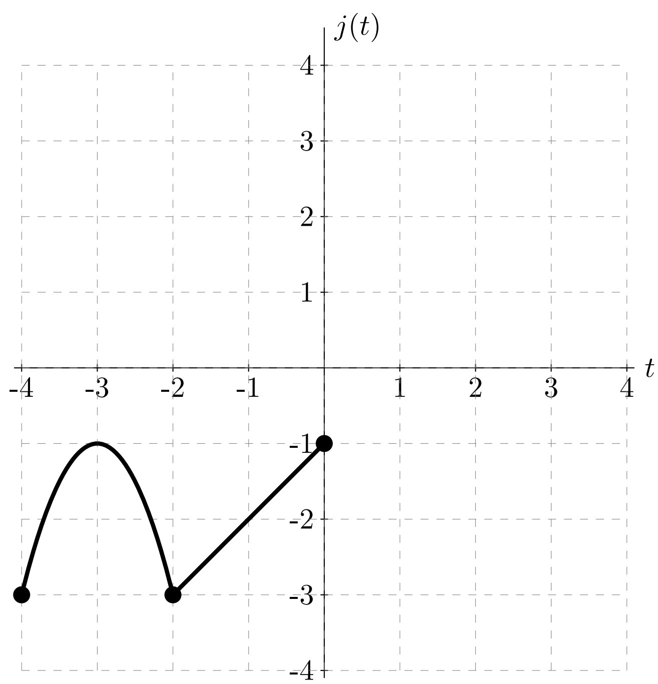
b. [4 points] Suppose $y = p(t)$ has vertical asymptote $t = 1$ and horizontal asymptote $y = 2$. Give the equations for a horizontal and vertical asymptote of the function $y = 2p(-t + 3) + 1$.

A horizontal asymptote of $2p(-t + 3) + 1$ is $\underline{\quad y = 5 \quad}$.

A vertical asymptote of $2p(-t + 3) + 1$ is $\underline{\quad t = 2 \quad}$.

c. [6 points] A graph of the function $h(t)$ is given below. On the empty set of axes, carefully sketch a well-labeled graph of $j(t) = -\frac{1}{2}h(t + 2) - 1$.





2. [11 points] The number of bees on Percy's uncle's farm has been decreasing over the past five years. The number of bees t years after 2012 on the farm is given by the exponential function

$$B(t) = 7000e^{-0.2t}.$$

a. [3 points] Find the annual decay rate of the bee population in exact form.

The annual decay rate is $\underline{\quad e^{-0.2} - 1 \quad}$.

Solution: If k is the continuous decay rate, and b is the growth factor, the annual decay rate is $r = b - 1 = e^k - 1 = e^{-0.2} - 1$.

b. [4 points] Percy's uncle will need to order more bees when the population of bees falls below 1000. How many years after 2012 will this occur? Give your answer in exact form or accurate to three decimal places.

Percy's uncle will need to order more bees $\underline{\quad \frac{\ln(1/7)}{-0.2} \quad}$ years after 2012.

Solution: Setting $B(t) = 1000$, we get

$$1000 = 7000e^{-0.2t}.$$

If we divide both sides by 7000 and then take ln of both sides, we get

$$\ln(1/7) = -0.2t.$$

So

$$t = \frac{\ln(1/7)}{-0.2}.$$

c. [4 points] The number of mosquitoes on Percy's uncle's farm has been increasing at an annual rate of 9%. Find the doubling time of the mosquito population. Give your answer in exact form or accurate to three decimal places.

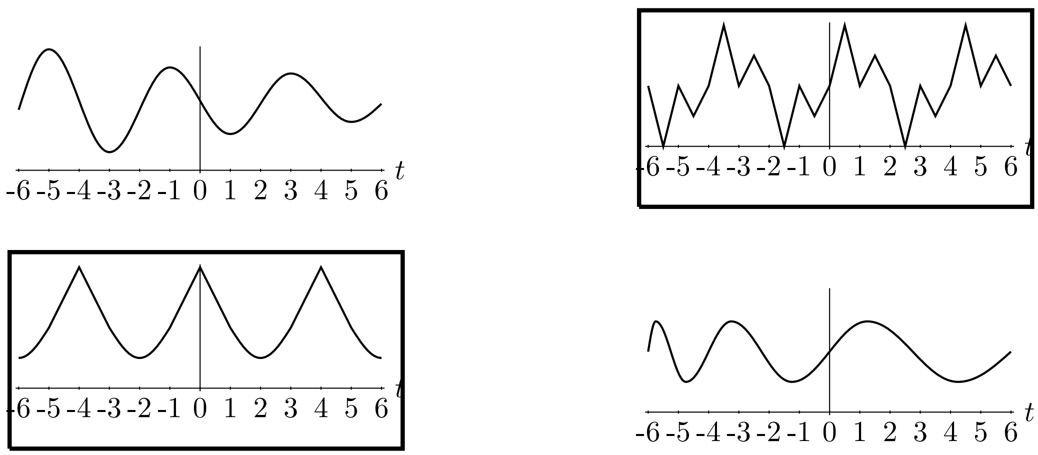
The doubling time of the mosquito population is $\underline{\frac{\ln 2}{\ln 1.09}}$ years.

Solution: The mosquito population can be modeled by an exponential function $M(t) = ab^t$ with t in years. If the annual growth rate is 9%, then $b = 1.09$. Using $2a$ for $M(t)$ to find doubling time, we get $2a = a(1.09)^t$. Solving, we get

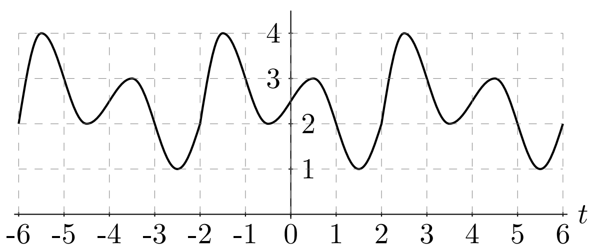
$$t = \frac{\ln 2}{\ln 1.09}.$$

3. [17 points]

a. [4 points] Circle all graphs in which the graphed function appears to be periodic with more than one period shown.



b. [2 points] Find the period of the function in the following graph:



The period is 4.

c. [5 points] Find the midline and amplitude of the function graphed in **b.**

The midline is y=2.5.

The amplitude is 1.5.

For parts **d.** and **e.** suppose $C(t)$ is the total number of calls received by a call center t hours after 8:00am on a normal day. Each sentence describes the number of calls the center receives on a particular day; circle the expression that corresponds to the given description.

d. [3 points] "The call center received 20 more calls than normal right at the beginning of the day, but otherwise it was a normal day."

$\boxed{C(t)+20}$ $\quad C(t+20)$ $\quad 20C(t)$ $\quad C(20t)$ $\quad$ None of these

e. [3 points] "The center was closed until noon, and at all times during the afternoon the call volume was twice what it normally would have been 4 hours earlier."

$2C(t+4)$ $\quad C(2t+8)$ $\quad C(2t+4)$ $\quad \boxed{2C(t-4)}$ $\quad$ None of these

4. [11 points] In chemistry, the pH of a substance is a function of the concentration of hydrogen ions per liter of the substance. The pH of a substance with concentration C hydrogen ions per liter is

$$A(C) = 23.78 - \log(C).$$

a. [2 points] Lemon juice has a pH of 2.28. What is the concentration of hydrogen ions per liter of lemon juice? Give your answer in exact form.

The concentration of hydrogen ions in lemon juice is $\underline{\quad 10^{21.5} \quad}$ ions per liter.

Solution: Start with $2.28 = 23.78 - \log(C)$. Then $\log(C) = 21.5$, so $C = 10^{21.5}$.

b. [4 points] If the number of hydrogen ions per liter C in a substance is doubled, what is the resulting change in pH? Write "increases" or "decreases" in the first blank, and the amount of increase or decrease in the second blank. Give your answer in exact form.

When the concentration of hydrogen ions in a substance doubles, the pH

decreases by $\log(2)$.

Solution: $A(2C) = 23.78 - \log(2C) = -\log 2 + (23.78 - \log(C)) = -\log 2 + A(C)$.

c. [2 points] The owner of the Peter and Sarah's regular pizza place is looking into canning her pizza sauce to sell in the supermarket. Currently her sauce has 10^{18} hydrogen ions per liter. What is the pH of her sauce? Give your answer in exact form or accurate to three decimal places.

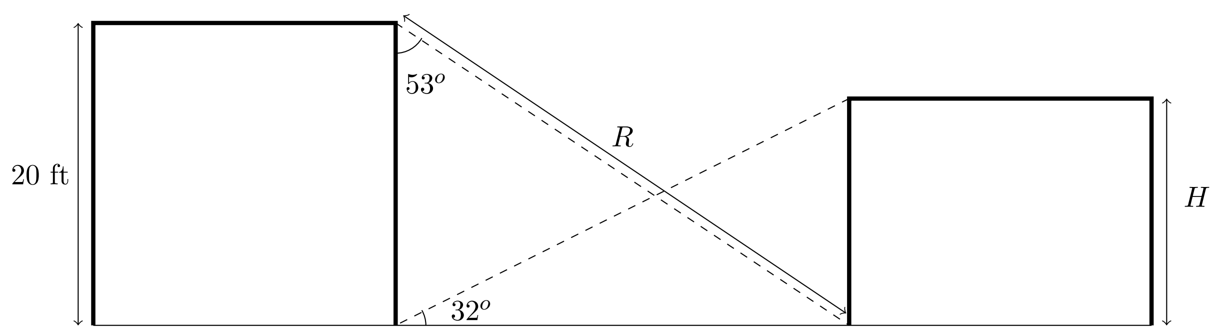
The pH of the sauce is 5.78.

d. [3 points] The state health department requires that the sauce have a pH lower than 4.7 in order for the sauce to be canned. How many times as many hydrogen ions per liter (compared to the current 10^{18}) will the sauce need in order for the health department to allow it to be canned? Give a whole number answer that results in a pH above 4 and below 4.7.

The sauce needs $\underline{\quad 16 \quad}$ times as many ions per liter to be canned.

Solution: Each time we double the number of hydrogen ions, the pH of the sauce decreases by $\log 2 \approx 0.3$ (actually it's a little more than 0.3) as we saw in part **b.** If we double the number of ions four times for 16 times the original number, the pH will fall by about 1.2 to about 4.58 which is in the range we want.

5. [8 points] Percy is building a zipline from the roof of his uncle's barn to the base of the farmhouse. The roof of the barn is 20 feet off of the ground. Looking at a 32 degree angle above the ground, he can see the roof of the farmhouse from the ground at the base of the barn. The line from the roof of the barn to the base of the farmhouse makes a 53 degree angle with the side of the barn. The situation is pictured below.



a. [3 points] Find R, the distance from the roof of the barn to the base of the farmhouse. Express your answer in exact form.

$R = \dfrac{20}{\cos 53}$.

Solution: $\cos 53 = 20/R$, so $R = \dfrac{20}{\cos 53}$.

b. [5 points] Find H, the height of the farmhouse. Express your answer in exact form. *(Hint: You may want to find the distance between the bases of the buildings first)*

$H = \dfrac{20 \tan 53}{\tan 58}$.

Solution: D, the distance between the buildings satisfies both

$$\frac{D}{20} = \tan 53$$

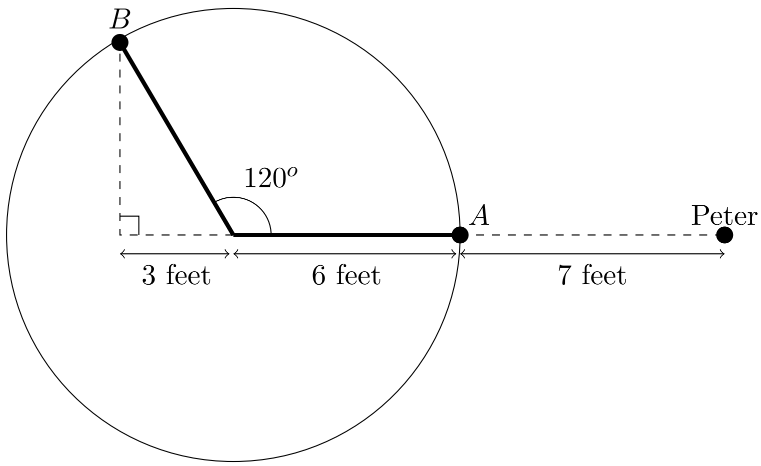
and

$$\frac{D}{H} = \tan 58.$$

Solving for D in both and setting them equal gives us $20 \tan 53 = H \tan 58$. So

$$H = \frac{20 \tan 53}{\tan 58}.$$

6. [15 points] At the park, Prem is riding on a merry-go-round of radius 6 feet spinning at a constant speed, and Peter is watching, 7 feet away from the merry-go-round. Prem starts at the point A and after 1.5 seconds he's at the point B. The situation is depicted below. The motion of the merry-go-round is counter-clockwise.



a. [2 points] How long does it take for the merry-go-round to complete one revolution?

It takes the merry-go-round ___4.5___ seconds to complete one revolution.

b. [2 points] How far did Prem travel along the circumference of the merry-go-round between point A and point B? Give your answer in exact form.

Prem traveled ___$6(\frac{2\pi}{3})$___ feet between point A and point B.

c. [2 points] By how many **radians** does the merry-go-round rotate in 3 seconds? Give your answer in exact form.

The merry-go-round rotates ___$\frac{4\pi}{3}$___ radians in 3 seconds.

d. [3 points] Find the distance between Peter and the point B.

The distance between Peter and the point B is ___$\sqrt{283}$___ feet.

e. [6 points] Find a function $D(\theta)$ that gives the distance in feet between Prem and Peter after Prem has rotated θ degrees from the point A.

$D(\theta) =$ ___$\sqrt{(13 - 6\cos(\theta))^2 + (6\sin(\theta))^2}$___ .

7. [8 points] At Peter and Sarah's regular pizza place, the pizza is 50 degrees Fahrenheit when it goes into the oven. The oven is 800 degrees Fahrenheit, so a pizza left in the oven will reach 800 degrees after a long time. After 6 minutes in the oven, the pizza is 200 degrees. The temperature of the pizza in degrees Fahrenheit after t minutes in the oven is a function of the form $P(t) = A + Be^{kt}$ with $k < 0$. Find the values of A, B and k in exact form. Show all of your work.

$A =$ 800

$B =$ -750

$k =$ $(\ln 4/5)/6$

Solution: When t is very large, Be^{kt} is very small, and $P(t)$ approaches 800, so $A = 800$.

$$P(0) = 50 = A + B,$$

so $B = -750$. Finally, we use the point $(6, 200)$ to get

$$200 = 800 - 750e^{6k}.$$

This means $4/5 = e^{6k}$, so $k = (\ln 4/5)/6$.

8. [7 points] The temperature in degrees Fahrenheit of the lasagna at the pizza place t minutes after it comes out of the oven is $L(t) = 75 + 225(0.9)^t$.

a. [2 points] What is the air temperature in the pizza place?

The air temperature in the pizza place is 75 degrees F .

b. [2 points] What is the temperature of the lasagna immediately after it comes out of the oven?

The temperature of the lasagna immediately after it comes out of the oven is 300 degrees F.

c. [3 points] How long after the lasagna comes out of the oven does it reach perfect eating temperature of 150 degrees Fahrenheit? Give your answer in exact form or accurate to three decimal places.

The lasagna reaches 150 degrees $\dfrac{\ln(1/3)}{\ln(0.9)}$ minutes after it comes out of the oven.

Solution: We start by setting

$$150 = 75 + 225(0.9)^t.$$

This means $(0.9)^t = 1/3$, so $t = \dfrac{\ln(1/3)}{\ln(0.9)}$.

9. [8 points] The following table gives values of several functions at different points. Use the table to answer the questions below.

t	-3	-2	-1	0	3	6
$X(t)$	-2	-1	-2	0	-2	-3
$Y(t)$	-3	-12	-1	-2	0	-2
$Z(t)$	-0.5	-3	-2	-3	9	12

a. [2 points] Could $X(t)$ be an odd function or an even function or can you be sure it's neither even nor odd? Circle your answer.

could be even | could be odd | couldn't be even or odd

b. [6 points] Which of the following transformations of $X(t)$ could be $Y(t)$, and which could be $Z(t)$? Write the letter(s) corresponding to your answers in the space provided. There could be more than one answer for each blank.

(A) $\frac{1}{2}X(3t+3)-2$

(B) $2X(-\frac{1}{3}t)+1$

(C) $X(-t+3)$

(D) $X(t-1)-1$

$Y(t)$ could be (C).

$Z(t)$ could be (A),(D).