Math 105 — First Midterm

March 20, 2018

UMID: EXAM SOLUTIONS Initials: ________

Instructor: ________ Section: ________

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1. **Do not open this exam until you are told to do so.**
2. **Do not write your name anywhere on this exam.**
3. This exam has 11 pages including this cover. There are 11 problems. Note that the problems are not of equal difficulty, so you may want to skip over and return to a problem on which you are stuck.
4. Do not separate the pages of this exam. If they do become separated, write your UMID on every page and point this out to your instructor when you hand in the exam.
5. Please read the instructions for each individual problem carefully. One of the skills being tested on this exam is your ability to interpret mathematical questions, so instructors will not answer questions about exam problems during the exam.
6. Show an appropriate amount of work (including appropriate explanation) for each problem so that graders can see not only your answer, but also how you obtained it. Include units in your answer where that is appropriate.
7. You may use a TI-84, TI-89, TI-Nspire or other approved calculator. However, you must show work for any calculation which we have learned how to do in this course.
8. If you use graphs or tables to find an answer, be sure to include an explanation and sketch of the graph, and to write out the entries of the table that you use.
9. **Turn off all cell phones, pagers, and smartwatches**, and remove all headphones.

Problem	Points	Score
1	12	
2	12	
3	10	
4	7	
5	7	
6	7	
7	8	
8	5	
9	10	
10	12	
11	10	
Total	100	

1. [12 points] In each of the following equations, solve for all possible values of x. Be sure to show your work and write your final answer in the blank in **exact** form. If there are no solutions, write "no solutions" in the blank.

a. [4 points] $\ln(2e^x - 5) = x$.

$x =$ $\ln 5$

Solution: Taking e to both sides, we get $2e^x - 5 = e^x$. After combining terms we have $e^x = 5$, so $x = \ln 5$.

b. [4 points] $e^{x+8} = 2^{7x-6}$.

$x =$ $\frac{8+6\ln 2}{7\ln 2-1}$

Solution: Applying ln to both sides and using properties of logs, we have

$$x + 8 = (7x - 6)\ln 2.$$

If we combine like terms, we get

$$(7\ln 2)x - x = 8 + 6\ln 2.$$

Factoring x out of the left hand side of the equation and dividing by what remains, we have that

$$x = \frac{8 + 6\ln 2}{7\ln 2 - 1}.$$

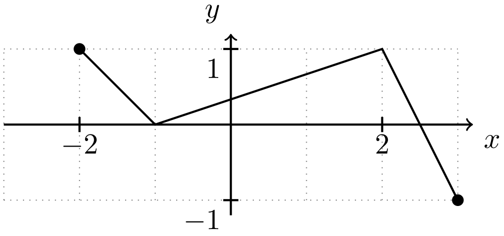
c. [4 points] $\log(2x^2 - 1) = 0$.

$x =$ ± 1

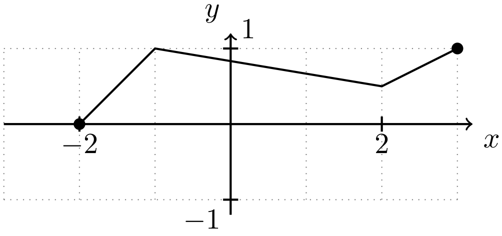
Solution: Taking 10 to both sides, we get $2x^2 - 1 = 1$. So $2x^2 = 2$, and $x = \pm 1$.

2. [12 points] Parts **a.** and **b.** of this problem are **unrelated** to each other.

a. [6 points] The graph of $y = f(x)$, defined on $[-2, 3]$ is given below.

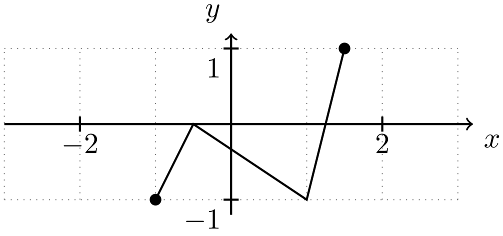


For each of the following two graphs, write a formula involving f that could give the graph.



This is the graph of

$y =$ $0.5f(-(x-1)) + 0.5$.



This is the graph of

$y =$ $-f(2x)$.

b. [6 points] If a function $f(x)$ has domain $[0, 3)$, range $[-1, \infty)$, and a vertical asymptote at $x = 3$, find the domain, range and vertical asymptote of the function

$$g(x) = \frac{1}{3} f(-x+1) - 2.$$

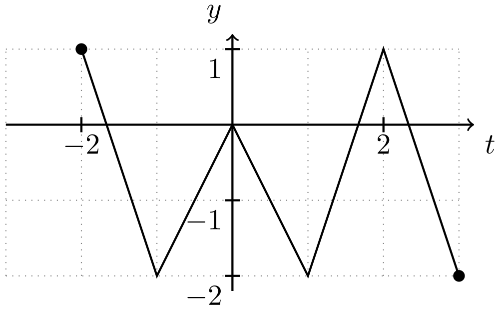
(i) The domain of $g(x)$ is $(-2, 1]$.

(ii) The range of $g(x)$ is $[-7/3, \infty)$.

(iii) The vertical asymptote of $g(x)$ is $x = -2$.

3. [10 points] Parts **a.** and **b.** of this problem are **unrelated** to each other.

a. [6 points] Part of the graph of a periodic function $y = N(t)$ is graphed below. The maximum and minimum value of $N(t)$ are shown.



Circle all of the following numbers that could be the period of $N(t)$? Unclear answers will be marked incorrect.

2 $\boxed{4}$ 5 $\boxed{6}$

Write the midline of $N(t)$ in the following blank: $y = -0.5$

Write the amplitude of $N(t)$ in the following blank: 1.5

b. [4 points] Suppose Fifi LeBlanc uses $F(E)$ feet of wrapping paper to package E chicken eggs. For the following practical interpretations, select the expression corresponding to the interpretation from the choices below and write your choice in the blank. You may find the following information useful: There are 12 inches in a foot, and there are 12 eggs in a dozen.

(i) The amount of wrapping paper, in **inches**, that Fifi uses to wrap d chicken eggs is

$12F(d)$.

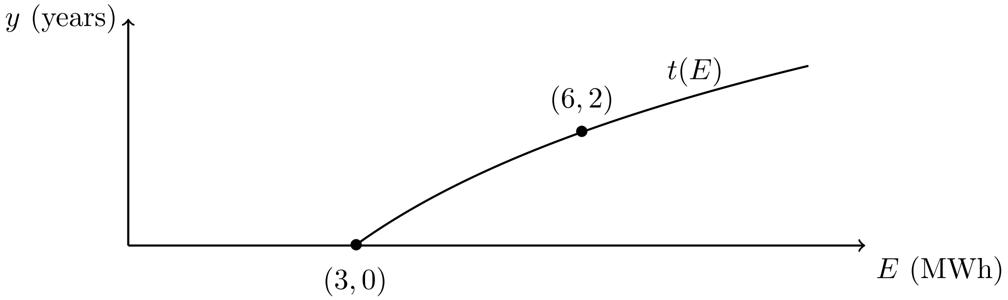
(ii) Fifi uses 12 times as much wrapping paper to wrap ostrich eggs as she does to wrap chicken eggs. Fifi uses $12F(12d)$ feet of wrapping paper to package d **dozen** ostrich eggs.

Possible answers:

$\frac{1}{12}F(12d)$ $12F(d)$ $12F(\frac{d}{12})$ $F(\frac{d}{12})$

$F(12d)$ $\frac{1}{12}F(d)$ $12F(12d)$ $\frac{1}{12}F(\frac{d}{12})$

4. [7 points] Mad scientist Kiki LeBlanc is analyzing the amount of energy she needs to run another one of her time machines named Machine1. Below is a graph of $y = t(E)$, the number of years into the past or future she can send a 1 kg notebook when the energy consumption of Machine1 is E megawatt-hours (MWh).



a. [4 points] The function $t(E)$ (assuming the domain is $E \geq 3$) can be written in the form $t(E) = a\log(E) + b$ for some constants a and b. Given the information in the graph, find a and b in **exact** form.

$a =$ $\frac{2}{\log 2}$.

$b =$ $-\frac{2\log 3}{\log 2}$.

Solution: If we use the two points on the graph, we get the system of equations:

$$0 = a\log(3) + b$$

$$2 = a\log(6) + b.$$

Solving for b in both equations and setting those expressions equal gives us

$$-a\log(3) = 2 - a\log(6).$$

If we solve for a we get

$$a = \frac{2}{\log 6 - \log 3} = \frac{2}{\log 2}.$$

This means $b = -\frac{2\log 3}{\log 2}$.

b. [3 points] Give a practical interpretation of the point $(6, 2)$ on the graph.

Solution: The point $(6, 2)$ on the graph means that, using Machine1, Kiki can send a 1kg notebook 2 years into the past or future when it's supplied with 6 MWh of energy.

5. [7 points] Another one of Kiki's time machines called Machine2 can send a 1kg notebook $y = r(E) = 2\log(E) - 3$ years into the past or future when it consumes E megawatt-hours (MWh) of energy.

a. [3 points] How much energy is required for the Machine2 to send a 1 kg notebook 5 years into the future? Be sure to show your work and give your answer in **exact** form with units.

10000 MWh of energy is required.

Solution: We need to fine E when $y = 5$, so we set $5 = 2\log(E) - 3$. Then $4 = \log(E)$ which means $E = 10^4$.

b. [4 points] Kiki has noticed that if she triples the energy input of Machine2, the number of years a 1 kg notebook travels in time increases by a fixed amount (that is not dependent on E). Find the amount of increase of $r(E)$ when E is tripled. Give your answer in **exact** form. Only solutions that show the amount of increase is not dependent on E will receive full credit.

$r(E)$ increases by $2\log(3)$ when E is tripled.

Solution: We subtract $r(3E) - r(E) = 2\log(3E) - 3 - (2\log(E) - 3) = 2\log(3)$.

6. [7 points] Kiki is experimenting with a highly radioactive substance, Isotope-Z. Isotope-Z decays at a (non-continuous) rate of 20% per day.

a. [3 points] Find the continuous decay rate of Isotope-Z. Give your answer as a **percentage** estimated to three decimal places or in exact form.

The continuous decay rate of Isotope-Z is $-100\ln(0.8)$.

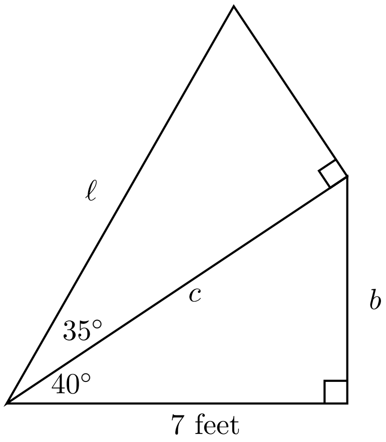
Solution: The daily growth factor for this substance is 0.8. The continuous decay rate is the natural log of the growth factor, so the continuous decay rate is $\ln(0.8)$. We multiply by 100 to make it a percentage, and we can leave it negative ($\ln(0.8) < 0$) or make it positive by adding a minus sign.

b. [4 points] Find the amount of time it takes for Isotope-Z to decrease to 60% of the initial amount present. Do not assume the initial amount is any specific number. Be sure to show all your work and leave your answer in **exact** form with units.

The amount of time is $\frac{\ln(0.6)}{\ln(0.8)}$.

Solution: If we call the initial amount A, we can set $0.6A = A(0.8)^t$. Then $0.6 = (0.8)^t$. Taking ln on both sides gives us $\ln(0.6) = t\ln(0.8)$ or $t = \frac{\ln(0.6)}{\ln(0.8)}$.

7. [8 points] Kiki is designing a sail for her new sailboat using two right triangles arranged as pictured in the figure below. The shared side between the triangles has length c.



Help Kiki by finding the lengths of b, c, and ℓ in **exact** form. None of your answers should include the letters b, c, or ℓ.

a. [2 points] $b =$ $7\tan(40°)$

b. [3 points] $c =$ $7/\cos(40°)$

c. [3 points] $\ell =$ $7/(\cos(40°)\cos(35°))$

8. [5 points] Suppose θ is an angle given in radians with $0 < \theta < \frac{\pi}{2}$ and with $\cos(\theta) = \frac{1}{3}$. Find the following in **exact** form (none of your answers should include the letter θ):

(i) $\sin(\theta) =$ $\sqrt{8}/3$

(ii) $\cos(\pi - \theta) =$ $-1/3$

9. [10 points] Hugo LeBlanc is baking bread. Suppose that t minutes after he put his bread in the oven, the temperature of the bread in degrees Fahrenheit is

$$y = B(t) = 350 - 290e^{-\frac{t}{7}}.$$

a. [2 points] Find the temperature of the bread when it is first put into the oven. Include units.

The temperature of the bread when it is first put into the oven is ______60° F______.

b. [2 points] Find $\lim_{t\to\infty} B(t)$.

$\lim_{t\to\infty} B(t) =$______350______.

c. [6 points] $B(t)$ is a transformation of the function e^t. Fill in the following blanks with the transformations needed to transform the graph of e^t into the graph of $B(t)$. On each line use one of the phrases given below for the first blank and a number for the second blank, if applicable (for reflections, do not use the second blank). Be sure to list the transformations in the proper order. Leave any unused lines blank.

SHIFT IT HORIZONTALLY TO THE RIGHT	SHIFT IT HORIZONTALLY TO THE LEFT	SHIFT IT VERTICALLY UPWARDS	SHIFT IT VERTICALLY DOWNWARDS	REFLECT IT OVER THE y-AXIS
COMPRESS IT HORIZONTALLY	STRETCH IT HORIZONTALLY	COMPRESS IT VERTICALLY	STRETCH IT VERTICALLY	REFLECT IT OVER THE t-AXIS

To get the graph of $B(t)$ starting with the graph of e^t,

first, we reflect it over the t-axis by ______,

and then we reflect it over the y-axis by ______,

and then we stretch it horizontally by ___7___,

and then we stretch it vertically by ___290___.

and then we shift it vertically upwards by ___350___.

10. [12 points] The following table gives values of several functions at different points. Use the table to answer the questions below.

t	-3	-2	-1	0	3	6
$A(t)$	-2	-1	-2	0	-2	-3
$B(t)$	-3	-1	-1	-3	3	-1
$C(t)$	0	-12	-1	0	0	2

a. [4 points] Could any of $A(t)$, $B(t)$, and $C(t)$ be an odd function or an even function or can you be sure any of them are neither even nor odd? Circle all that apply. Answer this part of the problem independently of your answers for the other parts.

Could be even: [✓ $A(t)$] $B(t)$ [✓ $C(t)$] None of these

Could be odd: $A(t)$ $B(t)$ [✓ $C(t)$] None of these

Isn't even or odd: $A(t)$ [✓ $B(t)$] $C(t)$ None of these

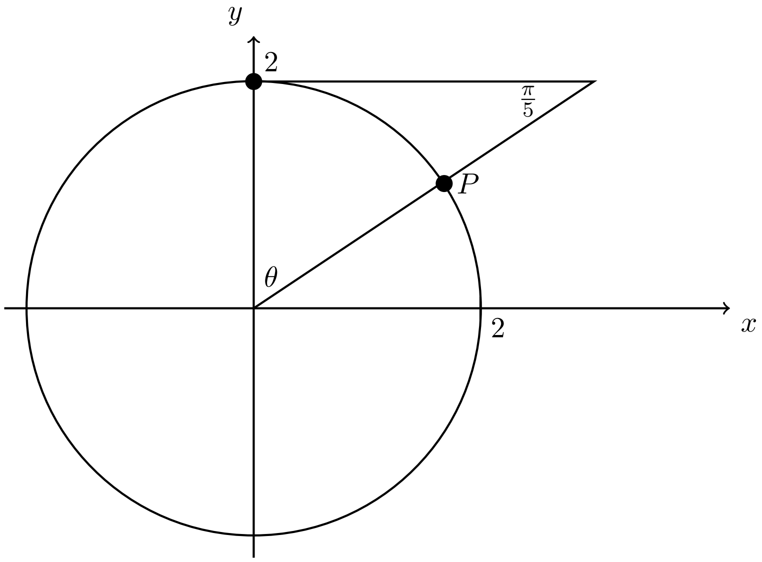
b. [3 points] Could any of the functions in the table be periodic with period 8? Circle all that apply Answer this part of the problem independently of your answers for the other parts.

Could have period 8: $A(t)$ [✓ $B(t)$] $C(t)$ None of these

c. [5 points] Circle all of the following transformations of $A(t)$ that **could** equal $B(t)$. If none of them could be $B(t)$, circle NONE.

- $\frac{1}{2}A(3t+3)-2$
- $2A(-\frac{1}{3}t)+1$
- [✓ $\frac{1}{2}A(t-9)$]
- NONE

11. [10 points] Consider the figure below in the xy-plane containing a circle of radius two and a right triangle with one angle measuring $\frac{\pi}{5}$ radians.



For all parts of this problem, express your answers in **exact** form.

a. [2 points] Find θ in radians.

$\theta =$ $3\pi/10$

b. [2 points] Find the length of the part of the circle **inside** the triangle between the point P and the point $(0, 2)$.

The length = $3\pi/5$

c. [2 points] Starting at the point $(2, 0)$ if we rotate 1 radian counterclockwise around the circle, is the resulting point inside the triangle, outside the triangle, or is it impossible to tell? Circle your answer.

[✓ inside the triangle] outside the triangle impossible to tell

d. [4 points] Find the $x-$ and $y-$coordinates of the point P.

$P = (\underline{2\cos(\pi/5)}, \underline{2\sin(\pi/5)})$