

Math 105 — First Midterm — March 10, 2021

1. This exam has 4 pages including this cover. There are 6 problems. Note that the problems are not of equal difficulty, so you may want to skip over and return to a problem on which you are stuck.
 2. Please read the instructions for each individual problem carefully. One of the skills being tested on this exam is your ability to interpret mathematical questions, so instructors will not answer mathematical questions about exam problems during the exam.
 3. Show an appropriate amount of work (including appropriate explanation) for each problem, so that graders can see not only your answer but how you obtained it. Partial credit will be awarded for correct work.
 4. Problems may ask for answers in *exact form*. Recall that $x = \frac{1}{3}$ is an exact answer to the equation $3x = 1$, but $x = 0.333$ is not.
 5. You do not need to “simplify” your answers unless asked to do so.
 6. You must use the methods learned in this course to solve all problems. Logarithm functions taught in this course include “log” (log base 10) and “ln” (natural log).
 7. You may use one pre-written page of notes, on an 8.5”x11” standard sheet of paper, with whatever you want written on both sides.
 8. You will not be allowed to use any other resources, including calculators, other notes, or the book.
 9. You must write your work and answers on **blank, white, physical paper**.
 10. You must write your **initials and UMID**, but not your name or unickname, in the upper right corner of every page of work. Make sure that it is visible in all scans or images you submit.
 11. Make sure that all pages of work have the relevant problem number clearly identified.
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Problem	Points
1	2
2	8
3	11
4	9
5	12
6	15
Total	57

1. [2 points] **There is work to submit for this problem. Read it carefully.**

- You may use your one pre-written page of notes, on an 8.5" by 11" standard sheet of paper, with whatever you want written on both sides.
- You are not allowed to use any other resources, including calculators, other notes, or the book.
- You may not use any electronic device or the internet, except to access the Zoom meeting for the exam, to access the exam file itself, to submit your work, or to report technological problems via the Google forms we will provide to do so. The one exception is that you may use headphones (e.g. for white noise) if you prefer, though please note that you need to be able to hear when the end of the exam is called in the Zoom meeting.
- You may not use help from any other individuals (other students, tutors, online help forums, etc.), and may not communicate with any other person other about the exam until **8am on Thursday** (Ann Arbor time).
- The one exception to the above policy is that you may contact the proctors in your exam room via the chat in Zoom if needed.
- Violation of any of the policies above may result in a score of zero for the exam, and, depending on the violation, may result in a failing grade in the course.

As your submission for this problem, you must write "I agree," and write your initials and UMID number to signify that you understand and agree to this policy. By doing this you are attesting that you have not violated this policy.

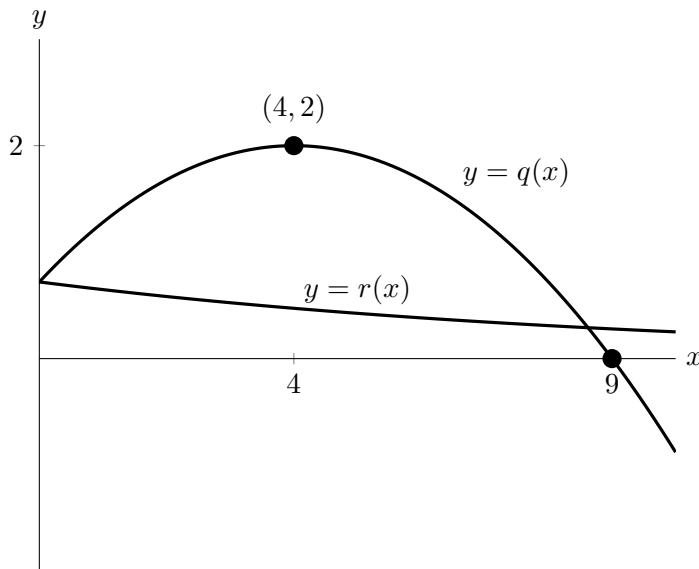
2. [8 points] Consider the following table of values for x , A , B , and C .

x	2	4	8	10
A	29	24	14	9
B	15.6	18.2	28.2	18.2
C	0	10	0	-3

For each of the following, decide whether the statement **could be true**. Briefly explain your reasoning.

- B is a function of A .
- A is a function of C .
- A is a linear function of x .
- If $C = f(x)$, then $f(x)$ is concave down.

3. [11 points] The graph below shows **part** of
- a quadratic function $q(x)$ with vertex and one zero marked
 - an exponential function $r(x) = ab^x$ that intersects $q(x)$ on the y -axis.



- a. [4 points] Find a formula for $q(x)$.
- b. [2 points] What is the x -coordinate of the other zero of $q(x)$?
- Recall that the formula for $r(x)$ is $r(x) = ab^x$. Use the graph and your formula for $q(x)$ to answer the following questions.
- c. [3 points] Which of the options below **could** be true? Briefly explain your answer.
- | | | |
|---------|-------------|---------|
| $a < 0$ | $0 < a < 1$ | $a > 1$ |
|---------|-------------|---------|
- d. [2 points] Which of the options below **could** be true? Briefly explain your answer.
- | | | |
|---------|-------------|---------|
| $b < 0$ | $0 < b < 1$ | $b > 1$ |
|---------|-------------|---------|
4. [9 points] An ice cream shop along the Huron river in Ann Arbor is only open in the summer. Its owner has designed a model that predicts the revenue (that is, the amount of money the shop takes in) of the shop in thousands of dollars, P , on a day where the maximum temperature is T degrees Fahrenheit. The model is described by the function $P = g(T)$, and has an inverse, $g^{-1}(P)$.
- The maximum temperature in Ann Arbor, in degrees Fahrenheit, on the d^{th} day that the shop is open for the summer, is given by the function $M(d)$.
- For each of the following, either give a practical interpretation of the given expression, or explain why the expression doesn't make sense in the context of the problem.
- a. [3 points] $g(M(13)) = 8$
- b. [3 points] $g^{-1}(5)$
- c. [3 points] $M(g^{-1}(7))$

5. [12 points] Jack is starting a business teaching others to paint. He has come up with the following pricing plan.
- For each lesson, a client has to pay a flat fee of \$6 to cover the cost of the art supplies they will use.
 - He charges \$2 per minute for the first 60 minutes of the lesson.
 - He charges \$0.50 per minute for each minute after that.
 - Each lesson lasts at most 120 minutes.

Let $C(m)$ be the amount of money he charges for a lesson that is m minutes long.

- a. [2 points] Evaluate $C(70)$.
- b. [6 points] Find a formula for $C(m)$. Use standard piecewise function notation:

$$C(m) = \left\{ \right.$$

- c. [4 points] The function $d = C(m)$, where d is the cost (in dollars) of a painting lesson that lasts m minutes, is invertible. Write a formula for its inverse $C^{-1}(d)$ using standard piecewise function notation.
6. [15 points] Scientists discover a new island in Lake Michigan and begin studying its animals. The island has both lizards and crows when they arrive, and they accidentally leave some mice on the island after discovering it.
- 5 thousand lizards live on the island when they discover it, but the population is decreasing at a rate of 5% per year.
 - Half a year after the island is discovered, the population of mice has grown to 2.3 times the initial population, and appears to be growing exponentially.
 - The population of crows, in thousands, t years after the island is discovered, can be modeled by $C(t) = 4e^{0.06t-1}$.

In the following problems, leave your answer in **exact form** and show every step of your work.

- a. [3 points] Find a formula for $L(t)$, the number of lizards on the island, in thousands, t years after the island is discovered.
- b. [3 points] How long does it take for the population of mice to reach 10 times the initial population?
- c. [2 points] What is the vertical intercept of $C(t)$? Interpret the meaning of this number in the context of the problem.
- d. [2 points] By what percentage does the population of crows increase in a year?
- e. [5 points] When will there be the same number of lizards and crows on the island?