Math 105 — Second Midterm — April 14, 2021

1. This exam has 4 pages including this cover. There are 6 problems. Note that the problems are not of equal difficulty, so you may want to skip over and return to a problem on which you are stuck.

2. Please read the instructions for each individual problem carefully. One of the skills being tested on this exam is your ability to interpret mathematical questions, so instructors will not answer mathematical questions about exam problems during the exam.

3. Show an appropriate amount of work (including appropriate explanation) for each problem, so that graders can see not only your answer but how you obtained it. Partial credit will be awarded for correct work.

4. Problems may ask for answers in *exact form*. Recall that $x = \frac{1}{3}$ is an exact answer to the equation $3x = 1$, but $x = 0.333$ is <u>not</u>.

5. You do not need to "simplify" your answers unless asked to do so.

6. You must use the methods learned in this course to solve all problems. Logarithm functions taught in this course include "log" (log base 10) and "ln" (natural log).

7. You may use one pre-written page of notes, on an 8.5"x11" standard sheet of paper, with whatever you want written on both sides.

8. You will not be allowed to use any other resources, including calculators, other notes, or the book.

9. You must write your work and answers on **blank, white, physical paper**.

10. You must write your **initials and UMID**, but <u>not</u> your name or uniqname, in the upper right corner of every page of work. Make sure that it is visible in all scans or images you submit.

11. Make sure that all pages of work have the relevant problem number clearly identified.

Problem	Points
1	2
2	10
3	14
4	11
5	12
6	11

Total	60

1. [2 points] **There <u>is</u> work to submit for this problem. Read it carefully.**

- You may use your one pre-written page of notes, on an 8.5" by 11" standard sheet of paper, with whatever you want written on both sides.

- You are <u>not</u> allowed to use any other resources, including calculators, other notes, or the book.

- You may <u>not</u> use any electronic device or the internet, except to access the Zoom meeting for the exam, to access the exam file itself, to submit your work, or to report technological problems via the Google forms we will provide to do so. The one exception is that you may use headphones (e.g. for white noise) if you prefer, though please note that you need to be able to hear when the end of the exam is called in the Zoom meeting.

- You may <u>not</u> use help from any other individuals (other students, tutors, online help forums, etc.), and may <u>not</u> communicate with any other person other about the exam until **11 am on Thursday** (Ann Arbor time).

- The one exception to the above policy is that you may contact the proctors in your exam room via the chat in Zoom if needed.

- Violation of any of the policies above may result in a score of zero for the exam, and, depending on the violation, may result in a failing grade in the course.

As your submission for this problem, you must write "I agree," and write your initials and UMID number to signify that you understand and agree to this policy. By doing this you are attesting that you have not violated this policy.

2. [10 points] Below are some values of functions $f(x), g(x)$, and $h(x)$.

| x	0	1	2	3	4
$f(x)$	2	0	4	4	3
$g(x)$	4	3	b	1	1
$h(x)$	3	a	3	0	0
$k(x)$	0	2	-3	1	0

Additionally:

- $h(x) = f(g(x))$

- The domain of $f(x)$ is $\{0, 1, 2, 3, 4\}$.

- $k(x)$ is an even, periodic function with period 10.

a. [6 points] Find the following values, or explain why they cannot be found from the given information. Be sure to show your work or explain your reasoning.

(i) a

(ii) b

(iii) $k(18)$

b. [4 points] Find all solutions to the equation $k(f(x)) = 0$.

3. [14 points]   Consider the function $f(x)$, graphed below. Note that $f(x)$ has one vertical asymptote and one horizontal asymptote, and $f(x)$ is not defined for x values to the left of those shown in the graph.
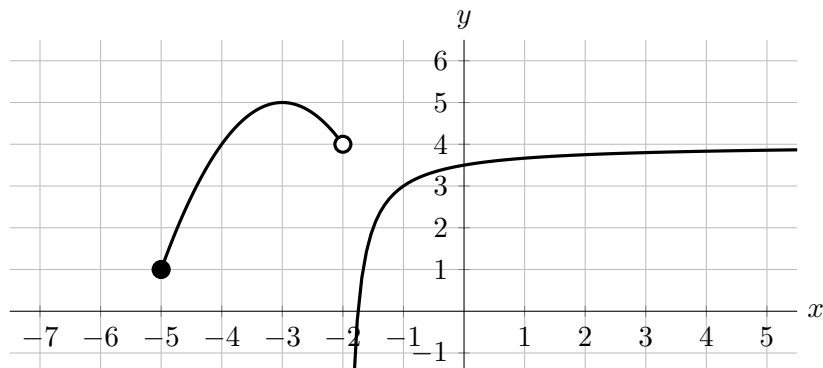

a. [7 points] Find:

(i) the domain of $f(x)$

(ii) the range of $f(x)$

(iii) an equation for the horizontal asymptote of $f(x)$

(iv) an equation for the vertical asymptote of $f(x)$

b. [7 points] Let $g(x) = 3f(-4(x-2)) + 1$. Find the following. Show how you obtained your answers, either by showing work, drawing diagrams, or explaining your reasoning.

(i) the domain of $g(x)$

(ii) an equation for the horizontal asymptote of $g(x)$

(iii) an equation for the vertical asymptote of $g(x)$

4. [11 points] Mia and Jonathan sell vegetables at the farmer's market at different booths. Their revenues, in **hundreds** of dollars, h hours after 9 am on a particular day are $M(h)$ (for Mia's revenue) and $J(h)$ (for Jonathan's revenue). Assume that the two functions are invertible.

a. [2 points] Give a practical interpretation of the equation $J(2) = 3$.

b. [3 points] Give a practical interpretation of the expression $J(M^{-1}(4))$, or explain why the expression does not make sense in the context of the problem.

c. [3 points] Write an equation corresponding to the following statement: Mia's revenue at 12pm is $100 less than twice Jonathan's revenue at 11 am.

d. [3 points] Let $T(k)$ be the total revenue, in **dollars** of both Mia and Jonathan k **minutes** after 9 am. Find a formula for $T(k)$ in terms of M and/or J.

5. [12 points] The graph of a sinusoidal function $y = K(t)$ is given below.
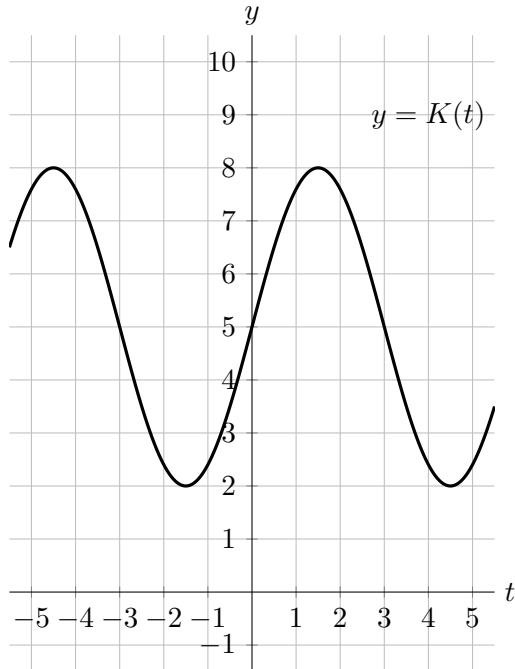
a. [7 points] Find the following.
  (i) The amplitude of $K(t)$.
  (ii) The midline of $K(t)$.
  (iii) The period of $K(t)$.
  (iv) A formula for $K(t)$.

b. [5 points] Find the first **three** positive values of t for which $K(t) = 7$. Give your answer in exact form.

6. [11 points] A duck in swimming in circles along the outer edge of a circular fountain in a park. The duck is 2 feet from the center of the fountain and swimming at a constant speed in a counter-clockwise direction. There is a sidewalk running north-south that passes 3 feet away from the fountain, as shown in the diagram below (which may not be drawn to scale). The duck starts at point A that is closest to the sidewalk. After 4 seconds, the duck is at the point B.
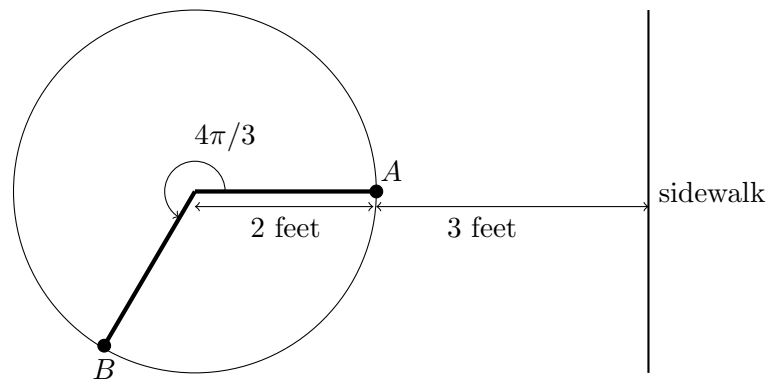
a. [2 points] How long does it take for the duck to make one full lap around the fountain? Include units.

b. [3 points] How far did the duck travel along the circumference of the fountain between point A and point B? Give your answer in exact form and include units.

c. [6 points] Find a function $D(t)$ that gives the (horizontal) distance in feet between the duck and the sidewalk t seconds after the duck starts swimming.