Math 105 — First Midterm — February 13, 2023

Write your 8-digit UMID number very clearly in the box to the right, and fill out the information on the lines below.

Your Initials Only: _____ Your 8-digit UMID number (not uniqname): _____

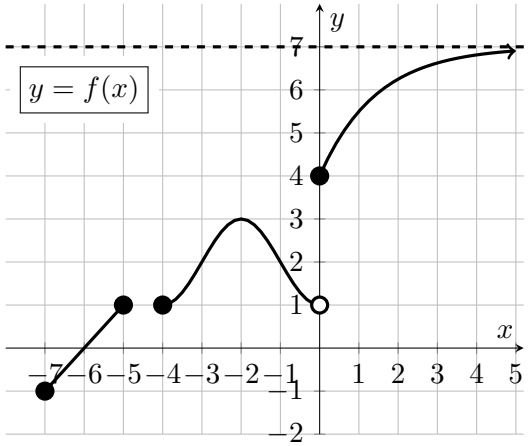
Instructor Name: _____ Section #: _____

1. **Do not open this exam until you are told to do so.**
2. **Do not write your name anywhere on this exam.**
3. This exam has 9 pages including this cover. There are 7 problems.
   Note that the problems are not of equal difficulty, so you may want to skip over and return to a problem on which you are stuck.
4. Do not separate the pages of this exam. If they do become separated, write your UMID (not name) on every page and point this out to your instructor when you hand in the exam.
5. The back of every page of the exam is blank, and, if needed, you may use this space for scratchwork. Clearly identify any of this work that you would like to have graded.
6. Read the instructions for each individual problem carefully. One of the skills being tested on this exam is your ability to interpret mathematical questions, so instructors will not answer questions about exam problems during the exam.
7. Show an appropriate amount of work for each problem, so that graders can see not only your answer but how you obtained it.
8. You must use the methods learned in this course to solve all problems.
9. You are allowed notes written on two sides of a $3'' \times 5''$ note card and one scientific calculator that does not have graphing or internet capabilities.
10. If you use a graph or table to find an answer, be sure to sketch the graph or write out the entries of the table. In either case, include an explanation of how the graph or table gives the answer.
11. Include units in your answer where that is appropriate.
12. Problems may ask for answers in *exact form*. Recall that $x = \sqrt{2}$ is a solution in exact form to the equation $x^2 = 2$, but $x = 1.41421356237$ is <u>not</u>.
13. **Turn off all cell phones, smartphones, and other electronic devices**, and remove all headphones, earbuds, and smartwatches. Put all of these items away. The use of any networked device while working on this exam is <u>not</u> permitted.

Problem	Points	Score
1	9	
2	8	
3	11	
4	11	

Problem	Points	Score
5	7	
6	6	
7	8	
Total	60	

1. [9 points] Part of the graph of a function $f(x)$ is shown below to the left; note that it has a **horizontal asymptote of** $y = 7$. Also shown is a table of some values for an invertible function $g(x)$, and formula for a function $h(x)$.



x	-5	-2	-1	0	1	2
$g(x)$	6	-5	0	4	7	9

$$h(x) = \begin{cases} x^2 + 1, & 0 \le x < \infty \\ x + 1, & -\infty < x < 0 \end{cases}$$

a. [3 points] Find the domain and range of $f(x)$. Give your answers using interval notation or using inequalities. *You do not need to explain or justify your answer.*

Domain: _____

Range: _____

b. [6 points] Find or estimate the value of each of the following; write N/A if a value does not exist or there is not enough information to find it.
You do not need to show work.

(i) $g(f(-1)) =$ _____

(ii) $f(g^{-1}(-5)) =$ _____

(iii) $h^{-1}(-5) =$ _____

(iv) $g(h(-2)) =$ _____

(v) $\lim_{x \to \infty} f(x) =$ _____

(vi) If $q(x) = g(x - 3) + 2$, $q(2) =$ _____

2. [8 points] The UM Etsy Club is 3D printing a new bracelet design called the Helix Monster. The cost of the materials for one bracelet depends on the inner circumference of that bracelet. The cost of materials B (in dollars) for a Helix Monster bracelet with an inner circumference of c centimeters is given by:

$$B = h(c) = 2 + 0.4c$$

a. [2 points] If the club members want to spend at most \$12 in materials on a Helix Monster bracelet, what is the largest the bracelet's inner circumference could be? *Include units.*

$c =$ _____

b. [3 points] Another member creates a Swirling Storm design that has different production costs. The cost (B, in dollars) to produce one Swirling Storm design with inner circumference c is given by

$$B = s(c) = 2.5 + 0.25c$$

For what values of c does the Helix Monster design cost less? For what values of c does the Swirling Storm design cost less? *Express your answers using inequalities or interval notation below. Show all work. No explanation needed.*

Helix Monster is cheaper when: _____

Swirling Storm is cheaper when: _____

c. [3 points] The club decides to produce a large batch of Swirling Storm bracelets with inner circumference 24cm. The price to rent the printer for the day is \$120. Write an expression for the total cost T (in dollars) for producing n Swirling Storm bracelets for inner circumference 24cm.

$T =$ _____

3. [11 points] The UM Dance Club met up with the UM Math Modeling Club to write formulas for different dancer's jumps. They measure one dancer's total time in the air as 1 second and their maximum height as 4 feet. They know that the function $D(t)$ which gives the dancer's height (in feet) as a function of time after they jump (in seconds) is a quadratic function.

a. [3 points] One member of the Math Modeling Club wants to find the formula for $D(t)$ using the zeros of the function, so is starting with the form:

$$D(t) = a(t - r)(t - s)$$

To model the dancer's jump described above, what are possible values of r and s and how do you know?

$r =$ _____

$s =$ _____

Explanation:

b. [3 points] Another member of the Math Modeling Club wants to write a formula using vertex form of a quadratic function:

$$D(t) = a(t - h)^2 + k$$

To model the dancer's jump described above, what are the values of h and k in this formula and how do you know?

$h =$ _____

$k =$ _____

Explanation:

The UM Dance Club met up with the UM Math Modeling Club to write formulas for different dancer's jumps. They measure one dancer's total time in the air as 1 second and their maximum height as 4 feet. They know that the function $D(t)$ which gives the dancer's height (in feet) as a function of time after they jump (in seconds) is a quadratic function.

c. [3 points] Find the exact value of a in the formulas above. *You can use either of your equations to do this. Show all work.*

$a =$_____

d. [2 points] From the context of the problem alone—without relying on or referring to your calculation above—would you expect the value of a to be positive or negative? Why?

$a > 0$ $a < 0$ NOT ENOUGH INFORMATION

Explanation:

4. [11 points] The UM Youtubers Club makes a very cool new video that goes viral. Suppose the video had 100 views at 2:00AM Eastern Time (ET) Saturday morning and the number of views grew exponentially for at least the next 24 hours, with views doubling every hour.

a. [1 point] Between 3:00AM ET and 6:00AM ET Saturday, by what factor had the number of views increased?

Factor of increase: _____

b. [2 points] Write a formula for a function $V = f(t)$, where V is the number of video views and t is the number of hours since 2:00AM ET Saturday.

$V = f(t) =$ _____

c. [6 points] For each of the following expressions or equations, explain its meaning in the context of the problem, or explain why it doesn't make sense in the context of the problem.

(i) $f^{-1}(500,000) \approx 12.25$

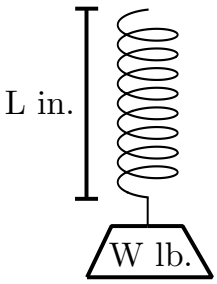
(ii) $\frac{f(5)-f(3)}{5-3} = 1200$

d. [2 points] Write a new function $g(s)$ **in terms of** f that would give us the number of views the video had s hours after 9:00AM ET on Saturday morning.

$g(s) =$ _____

5. [7 points] The UM Weights and Measures Club is building a spring scale, which weighs objects by hanging them from a spring.

Let W be the weight of an object, in pounds, and let L be the length of the spring in inches when we hang that object from it. It turns out that there is a linear relationship between W and L. The club observes that their spring is 3 inches long when no weight is attached, and stretches out to 5.5 inches when they test it with a 5-pound weight.



a. [3 points] What is the slope of the function $W = f(L)$? Explain the meaning of the slope's value in the context of the problem.

Slope = _____

Meaning:

b. [2 points] Find a formula for $W = f(L)$.

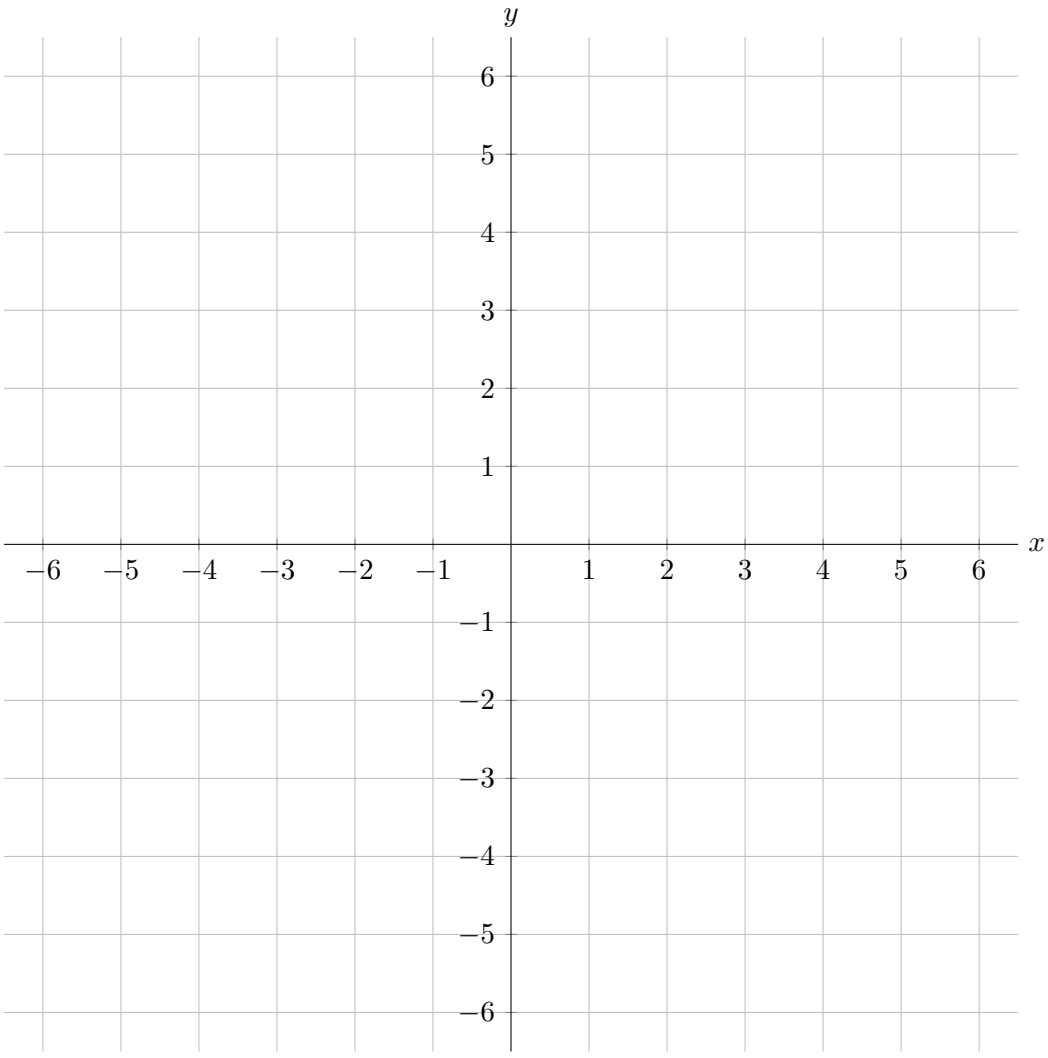
$W =$ _____

c. [2 points] Suppose we hang a bucket from the spring and then pour in some water. As we add the weight of the water, the spring gets 4 inches longer. How much does the added water weigh? *Include units.*

The water in the bucket weighs _____

6. [6 points] On the axes below, sketch the graph of a single function $y = f(x)$ with all of the following properties:

- The domain of $f(x)$ is $-5 \leq x < \infty$.
- The range of $f(x)$ is $-3 \leq x < 3$.
- $f(x)$ is concave down on the interval $-5 \leq x \leq -3$.
- $f(x)$ is decreasing on the interval $-4 \leq x \leq -1$.
- The average rate of change of $f(x)$ between $x = -1$ and $x = 3$ is $1/2$.
- $f(x) \to 3$ as $x \to \infty$



7. [8 points] The following table shows some values of 3 different functions:

x	6	7	9
$f(x)$	27	18	8
$g(x)$	15	20	25
$h(x)$	12	16	24

This blank space is for your work and calculations.

Circle **all** correct options for each part.

a. [2 points] Which of these functions could be linear?

$f(x)$ $g(x)$ $h(x)$ NONE OF THESE

b. [2 points] Which of these functions could be exponential?

$f(x)$ $g(x)$ $h(x)$ NONE OF THESE

c. [2 points] Which of these functions could be concave up on the interval $[6, 9]$?

$f(x)$ $g(x)$ $h(x)$ NONE OF THESE

d. [2 points] Which of these functions could be increasing on the interval $[6, 9]$?

$f(x)$ $g(x)$ $h(x)$ NONE OF THESE