Math 105 — Second Midterm — March 20, 2023

Write your 8-digit UMID number very clearly in the box to the right, and fill out the information on the lines below.

Your Initials Only: _____ Your 8-digit UMID number (not uniqname): _____

Instructor Name: _____ Section #: _____

1. **Do not open this exam until you are told to do so.**
2. **Do not write your name anywhere on this exam.**
3. This exam has 10 pages including this cover. There are 7 problems.
   Note that the problems are not of equal difficulty, so you may want to skip over and return to a problem on which you are stuck.
4. Do not separate the pages of this exam. If they do become separated, write your UMID (not name) on every page and point this out to your instructor when you hand in the exam.
5. The back of every page of the exam is blank, and, if needed, you may use this space for scratchwork. Clearly identify any of this work that you would like to have graded.
6. Read the instructions for each individual problem carefully. One of the skills being tested on this exam is your ability to interpret mathematical questions, so instructors will not answer questions about exam problems during the exam.
7. Show an appropriate amount of work for each problem, so that graders can see not only your answer but how you obtained it.
8. You must use the methods learned in this course to solve all problems.
9. You are allowed notes written on two sides of a $3'' \times 5''$ note card and one scientific calculator that does not have graphing or internet capabilities.
10. If you use a graph or table to find an answer, be sure to sketch the graph or write out the entries of the table. In either case, include an explanation of how the graph or table gives the answer.
11. Include units in your answer where that is appropriate.
12. Problems may ask for answers in *exact form*. Recall that $x = \sqrt{2}$ is a solution in exact form to the equation $x^2 = 2$, but $x = 1.41421356237$ is <u>not</u>.
13. **Turn off all cell phones, smartphones, and other electronic devices**, and remove all headphones, earbuds, and smartwatches. Put all of these items away. The use of any networked device while working on this exam is <u>not</u> permitted.

Problem	Points	Score
1	10	
2	10	
3	11	
4	11	

Problem	Points	Score
5	6	
6	7	
7	5	
Total	60	

1. [10 points] Below is a table giving some values of an **odd** function $f(x)$. The domain of $f(x)$ is $(-\infty, \infty)$ (all real numbers).

x	2	3	4	5
$f(x)$	-3	-1	-1	1

a. [3 points] Find the following values of f, or write NEI if there is "not enough information" to find the value.

(i) $f(-2) =$ _____

(ii) $f(1) =$ _____

(iii) $f(0) =$ _____

b. [2 points] Could f be an invertible function? Explain your answer.

The function f (*circle one*): COULD BE INVERTIBLE COULD NOT BE INVERTIBLE

Explanation:

c. [4 points] Recall that $f(x)$ is an odd function. For each of the following functions, decide whether it is even, odd, neither, or if there is not enough information (NEI) to tell. *No explanation needed.*

(i) The function $g(x) = x^3 f(x)$ is (*circle all that apply*):

ODD EVEN NEITHER NEI

(ii) The function $h(x) = x^2 + f(x)$ is (*circle all that apply*):

ODD EVEN NEITHER NEI

d. [1 point] Suppose it is also true that: $\lim_{x\to\infty} f(x) = 5$. Use this information to find $\lim_{x\to-\infty} f(x)$, or write NEI if there is not enough information to find the limit.

$\lim_{x\to-\infty} f(x) =$ _____

2. [10 points] The Executive Director of the Go Blue Zoo is managing the zoo's finances. She tracks the following functions:

- $O(n)$ is the number of pounds of fish per day used to feed n otters.
- $P(m)$ is the number of pounds of fish per day used to feed m penguins.
- $F(s)$ is the cost, in dollars, of s pounds of fish.

a. [6 points] For each of the following, describe the meaning of the expression or equation in the director's context, or explain why the expression or equation doesn't make sense.

(i) $O(6) = 70$

(ii) $O^{-1}(P(8))$

(iii) $P(10) + F(10)$

b. [4 points] For each of the following statements or quantities, write an equivalent mathematical expression or equation, or explain why there isn't enough information to do so.

(i) To feed its 5 otters and 8 penguins, the zoo uses 86 pounds of fish per day.

Expression or Equation: _____

(ii) The daily cost, in dollars, of feeding 5 otters.

Expression or Equation: _____

3. [11 points] The Go Blue Zoo's power has gone out and it is cold outside! The indoor frog exhibit is typically kept warm, but it is now getting colder. The exhibit temperature, in °F, t hours after the power goes out is given by:

$$E(t) = 30 + we^{kt}, \quad \text{where } w \text{ and } k \text{ are constants.}$$

a. [3 points] When the power first went out, the temperature of the frog exhibit was $75°F$, but after 4 hours the temperature is 68°F. Find the values of w and k.
Show all work. Give your answers in exact form, or accurate to two decimal places.

$w =$ _____ $k =$ _____

b. [2 points] What is $\lim_{t\to\infty} E(t)$? Explain what this number means in the context of the problem.

$\lim_{t\to\infty} E(t) =$ _____

Meaning:

c. [4 points] Last summer the Go Blue Zoo also had a power outage. This time it was hot outside and the refrigerator for storing the tiger's food started warming up. After t hours, the temperature inside the refrigerator, in $°F$, was given by

$$R(t) = 75 - 38e^{-0.03t}$$

Due to safety concerns, food must be thrown out if the temperature inside a refrigerator rises above 40°F. How long could the power outage last without having to throw out the tiger's food? *Show all work. Give your answer in exact form, or accurate to two decimal places.*

_____ hours

This problem continues on the next page.

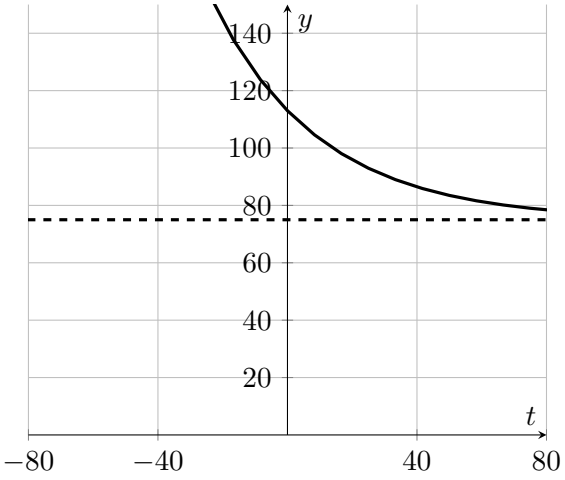
d. [2 points] Recall that we're considering the function

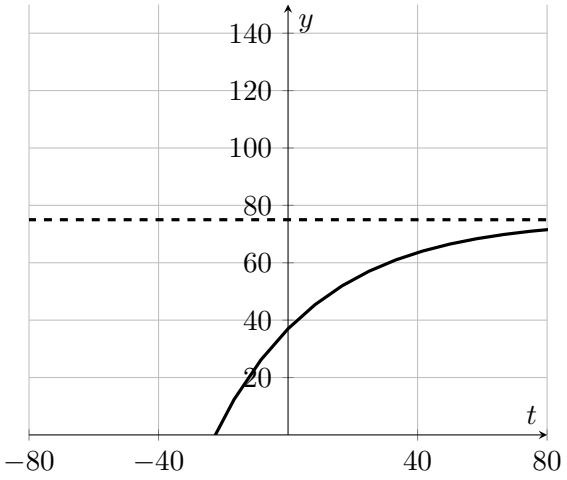
$$R(t) = 75 - 38e^{-0.03t}$$

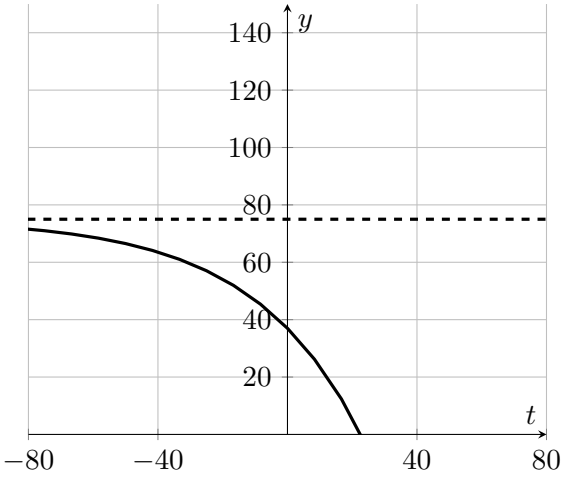
which tracks the temperature inside a refrigerator, in $^\circ F$, t hours after the power goes out on a hot day.

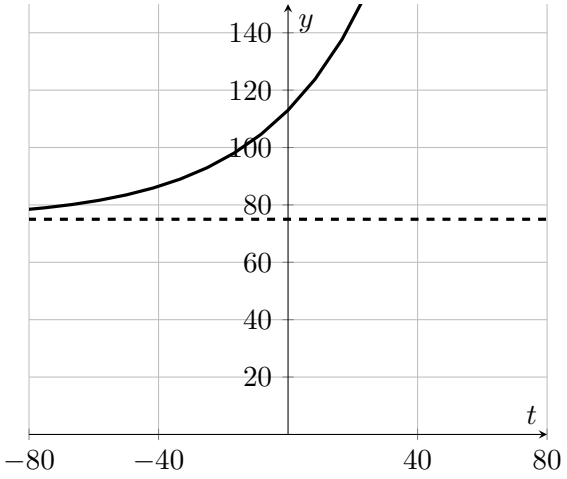
The domain of that function in *this* context would be $t \geq 0$; however, to help you make sense of meaningful features, the graphs below are shown on a larger domain.

Which of the four graphs show below could be the graph of $R(t)$? (*Circle one*)





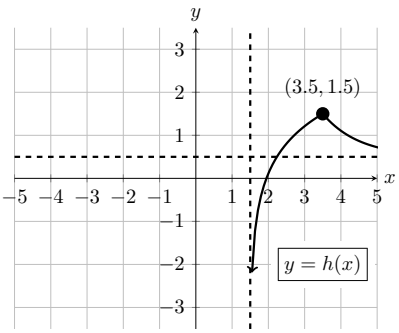




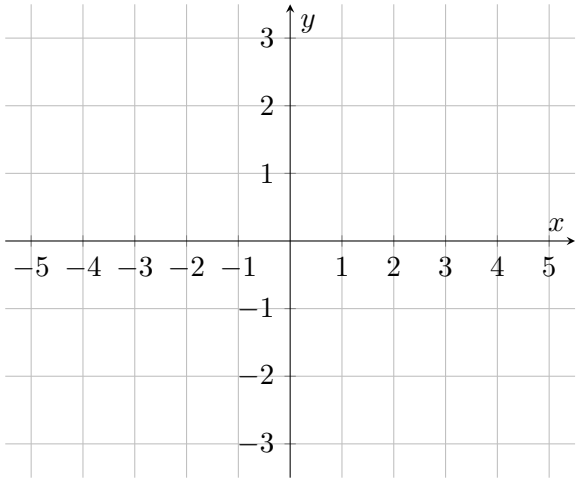
4. [11 points]

a. [6 points]

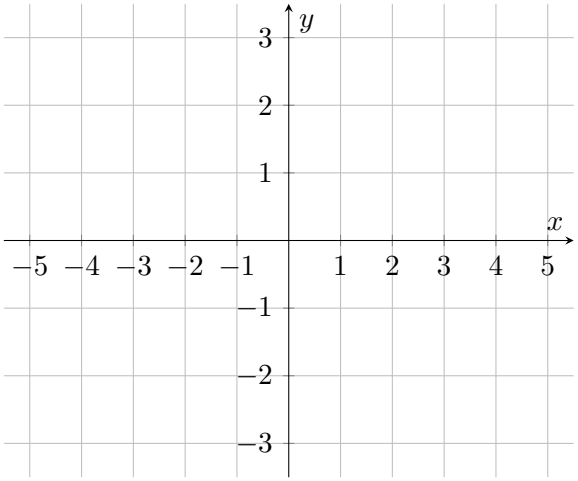
To the right is a graph of a function $h(x)$. The graph of $h(x)$ goes through the corner point $(3.5, 1.5)$, has a horizontal asymptote at $y = 0.5$, and a vertical asymptote at $x = 1.5$. We will apply a sequence of transformations to the graph of $h(x)$. For each subsequent transformation, sketch the intermediate graphical result on the given set of axes. **For each step, clearly denote any asymptotes with a dotted line, and label coordinates of the graph's corner point.**



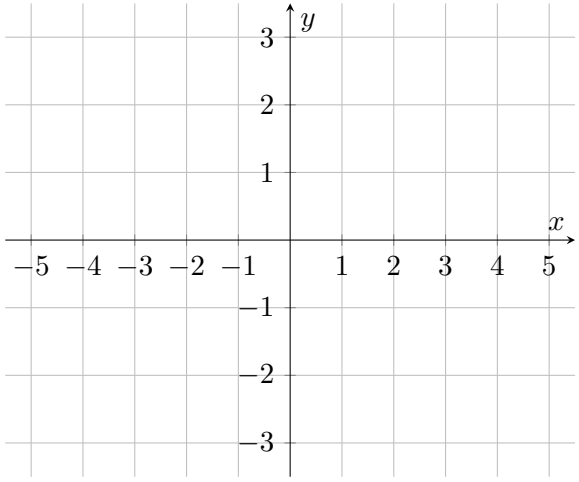
Step 1: Reflect the graph of $h(x)$ over the y-axis.



Step 2: Stretch the graph from Step 1 vertically by a factor of 2.



Step 3: Shift the graph from Step 2 right by 4.



This problem continues on the next page.

b. [3 points] Consider the function represented by the final graph in Step 3 and call it $g(x)$. Give a formula for $g(x)$ in terms of the original function $h(x)$.

$g(x) =$ _____

c. [2 points] Use the final coordinates of your corner point in the graph of $g(x)$ (that is, Step 3) to check if your formula in part (b) is correct. *You can get points for this part of the problem even if your formula above is incorrect and you figure that out in this step, but don't know how to correct your formula.*

5. [6 points] For fun, Booboo the chimpanzee likes to climb up a pole in the Ape House at the Go Blue Zoo and drop his doll Deedee. The function $T(h)$ gives the number of seconds it takes for Deedee to hit the ground when Booboo drops her from h feet above ground level. Write an expression for each of the following new functions, in terms of the function T:

a. [2 points] $R(h)$ is the time it takes, *in minutes*, for Deedee to hit the ground when dropped from h feet above ground level. (Note that 1 minute is equivalent to 60 seconds.)

$R(h) =$ _____

b. [2 points] $F(y)$ is the time it takes, in seconds, for Deedee to hit the ground when dropped from y *yards* above ground level. (Note that 1 yard is equivalent to 3 feet.)

$F(y) =$ _____

c. [2 points] There is a small platform mounted on the pole 8 feet up from the ground. Sometimes Deedee lands there instead of the ground. The function $P(h)$ gives the time, in seconds, for Deedee to hit the platform when dropped from h feet above ground level. If we want to express $P(h)$ in terms of the function T, circle the best option below.

$P(h) = T(h+8)$ $\quad$ $P(h) = T(h-8)$ $\quad$ $P(h) = T(h)+8$ $\quad$ $P(h) = T(h)-8$

6. [7 points] The Go Blue Zoo breeds its own insect populations for feeding some of the birds and reptiles. However, a virus infected the cricket breeding program on January 1, 2023. The cricket population started at 1,000,000 crickets, but decreased by 15% per week once the colony was infected.

a. [3 points] How many weeks did it take for the population to decrease by 60%? *Show all work. Give your answer in exact form, or accurate to at least two decimal places.*

_____ weeks

b. [2 points] By what percentage did the cricket population decrease for each *day*? *Show all work. Give your answer in exact form, or accurate to at least two decimal places.*

Decreases by _____ % per day.

c. [2 points] If the population instead decayed at a *continuous* rate of 15% per week, by what *non-continuous* percentage would it decrease in one week? *Show all work. Give your answer in exact form, or accurate to at least two decimal places.*

Decreases by _____ % per week.

7. [5 points]

a. [3 points] Let k, j be positive constants with $k > j > 0$. If any of the following values are undefined, write them in the "Undefined:" blank below. Then sort the remaining values from least to greatest.

0 $\log(-k)$ $\log(k/j)$ $\log(j/k)$ $\log(0)$

Undefined: _____

Remaining quantities ordered from LEAST to GREATEST:

_____

b. [2 points] If $\log(ab) = 2$, then...

Give a value for each of the following, or write NEI *if there is not enough information to answer.*

$ab =$ _____

$\log(a) \cdot \log(b) =$ _____