Math 105 — Final Exam — April 24, 2023

Write your 8-digit UMID number very clearly in the box to the right, and fill out the information on the lines below.

Your Initials Only: _____ Your 8-digit UMID number (not uniqname): _____

Instructor Name: _____ Section #: _____

1. **Do not open this exam until you are told to do so.**
2. **Do not write your name anywhere on this exam.**
3. Use a pencil for "bubble-in" questions so that you can easily erase your answer if you change your mind.
4. This exam has 11 pages including this cover. There are 8 problems. Note that the problems are not of equal difficulty, so you may want to skip over and return to a problem on which you are stuck.
5. Do not separate the pages of this exam. If they do become separated, write your UMID (not name) on every page and point this out to your instructor when you hand in the exam.
6. The back of every page of the exam is blank, and, if needed, you may use this space for scratchwork. Clearly identify any of this work that you would like to have graded.
7. Read the instructions for each individual problem carefully. One of the skills being tested on this exam is your ability to interpret mathematical questions, so instructors will not answer questions about exam problems during the exam.
8. Show an appropriate amount of work for each problem, so that graders can see not only your answer but how you obtained it.
9. You must use the methods learned in this course to solve all problems.
10. You are allowed notes written on two sides of a $3'' \times 5''$ note card and one scientific calculator that does not have graphing or internet capabilities.
11. If you use a graph or table to find an answer, be sure to sketch the graph or write out the entries of the table. In either case, include an explanation of how the graph or table gives the answer.
12. Include units in your answer where that is appropriate.
13. Problems may ask for answers in *exact form* or in *decimal form*. Recall that $\sqrt{2} + \cos(3)$ is in exact form and $x = 0.424$ would be the same answer expressed in decimal form.
14. **Turn off all cell phones, smartphones, and other electronic devices**, and remove all headphones, earbuds, and smartwatches. Put all of these items away. The use of any networked device while working on this exam is <u>not</u> permitted.

Problem	Points	Score
1	8	
2	7	
3	8	
4	7	

Problem	Points	Score
5	5	
6	10	
7	10	
8	5	
Total	60	

1. [8 points]

a. [3 points] Em, an employee at the *Math-tas-tíque Dog Boutique*, earns \$750 per week in salary and earns an additional 5% of her total sales that week (her *commission*). Write a formula for $M(x)$, the amount, in dollars, Em will earn in a week in which she is responsible for $\$x$ in sales.

$M(x) =$ _____

b. [3 points] Compute the value of $M^{-1}(1000)$ and describe its meaning in the context of the problem.
Show all work. Give your final answer in decimal form, NOT exact form.

$M^{-1}(1000) =$ _____

Meaning:

c. [2 points] Let $R(w)$ be the function giving the dollar amount of Em's sales in the wth week of 2023. Choose the best description of the meaning of $M(R(23))$ from the choices below.

◯ A. The week in which Em makes \$23 in commission.

◯ B. The amount of commission Em makes in the 23rd week of 2023.

◯ C. The total amount Em gets paid in 2023.

◯ D. The total amount Em gets paid in in the 23rd week of 2023.

◯ E. This doesn't make sense because we cannot plug a number of weeks into the function M.

2. [7 points]

a. [4 points] A population of fleas takes residence at the nearby *I-Love-Functions Dog Hotel* (oh no!) and the population grows exponentially for the first couple of days. At $t = 2$ hours after the infestation started, the population is 1000 fleas. By $t = 6$ hours after it started, the population is 2000 fleas. Write a formula for $P(t)$, the number of fleas t hours after the infestation started.
Show all work. Your final formula should include parameters in their EXACT form.

$P(t) =$ _____

b. [3 points] *Last* year a population of fleas also took up residence at the hotel and their population, as a function of hours since their arrival, was given by:

$$Q(t) = 500(1.22^t)$$

By what percent did *this* population increase each hour?

_____ %

How long did it take for their initial population to triple?
Show all work. Give your final answer in decimal form, NOT exact form.

_____ hours

3. [8 points] Traditionally, it has been assumed that a D year-old dog is the same biological age as a $7D$ year-old human. So a 3 year-old dog (in actual years) has aged as much as a 21 year-old human.

However, scientists have found a new aging formula for Labrador retrievers that takes specific biological aging markers into account. The new formula claims that a D year-old Labrador retriever (in actual years) has aged as much as a human who is

$$H = f(D) = 15\ln(D) + 31 \text{ years old}$$

One strange thing about this formula they came up with is that it doesn't go through the point $(0, 0)$ as we'd expect it to. In fact, we can't plug in 0 to this formula at all!

a. [2 points] Explain in one sentence why we can't plug $D = 0$ into this formula.

Explanation:

b. [3 points] According to this formula, at what age (in real years) will a dog be biologically equivalent to a newborn baby ($H = 0$)?
Show all work. Give your final answer in decimal form, NOT exact form.

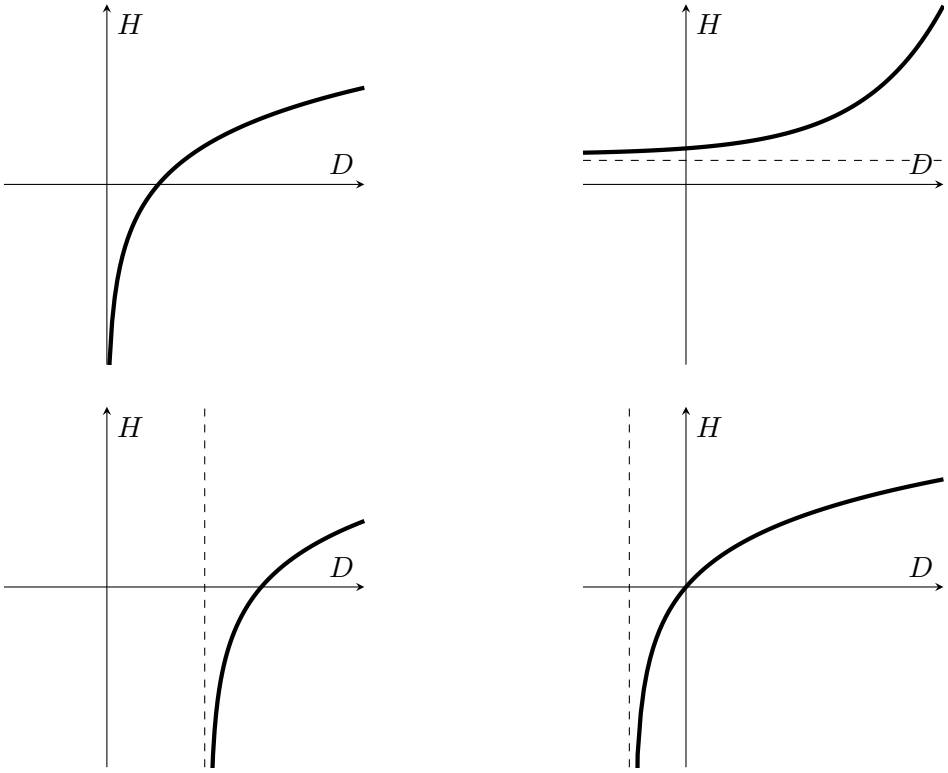
$D =$_____years

This problem continues on the next page.

c. [3 points] Now considering the same function without its context: which of the graphs below could be the graph of

$$f(D) = 15\ln(D) + 31?$$

Circle the correct graph or NONE.



NONE OF THESE GRAPHS COULD REPRESENT THE FUNCTION $f(D)$.

4. [7 points] Dog owner Malik recently bought an Extra-High-Flying Ball$^{\text{TM}}$ at the *Math-tas-tíque Dog Boutique* for his extra-high-jumping Jack Rusell Terrier.

On one particular throw, the ball's height, in feet, is given by:

$$h(t) = -16(t + \tfrac{1}{8})(t - 3),$$

where t is the number of seconds after the ball left Malik's hand.

a. [2 points] At what height was the ball when it was released Malik's hand?
Show all work. Give your final answer in decimal form, NOT exact form.

height: _____ feet

b. [3 points] What is the maximum height the ball reached and at what time did it reach that height?
Show all work. Give your final answer in decimal form, NOT exact form.

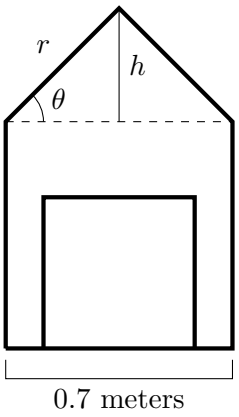
height: _____ feet

time: _____ seconds

c. [2 points] Assuming the ball wasn't caught on its way down, how many seconds, total, was the ball in the air?

time in the air: _____ seconds

5. [5 points] Another customer of the dog boutique is making a custom dog house. A sketch of their plans (not drawn to scale) is shown below:



a. [2 points] In order to for the snow to slide off, the slope of the roof should rise at least 4 inches vertically for each 12 inches in horizontal change. **If $\theta = 20°$ will the roof be steep enough for snow to slide off?** *Show all expressions that you calculate.*

(Circle one) YES NO NOT ENOUGH INFORMATION

b. [3 points] The dog owner decides to make $\theta = 22°$. If the overall width of the front piece shown is 0.7 meters, what will be the measurement of r shown in the diagram?
Show all work. Give your final answer in decimal form, NOT exact form.

$r =$_____meters

6. [10 points] Color in the blank circle for **all possible** correct choices. *Remember to use pencil so that you can erase your answers if you change your mind!*

a. [2 points] A graph goes through the points $(1, 2)$ and $(-1, 6)$.

This graph *could* represent a(n) _____ function.

- ◯ linear
- ◯ exponential
- ◯ periodic
- ◯ odd
- ◯ NONE OF THE ABOVE

b. [2 points] A graph goes through the points $(2, 4)$ and $(2, 10)$.

This graph could represent a(n) _____ function.

- ◯ linear
- ◯ exponential
- ◯ periodic
- ◯ odd
- ◯ NONE OF THE ABOVE

This problem continues on the next page.

c. [2 points] $f(x) = 4(x-2) + 3x + 8$.

$f(x)$ is a(n) _____ function.

- ◯ linear
- ◯ exponential
- ◯ periodic
- ◯ odd
- ◯ NONE OF THE ABOVE

d. [2 points] $g(x) = e^{3(x-4)}$.

$g(x)$ is a(n) _____ function.

- ◯ linear
- ◯ exponential
- ◯ periodic
- ◯ odd
- ◯ NONE OF THE ABOVE

e. [2 points] $h(x) = \frac{2}{3}\sin(4x)$

$h(x)$ is a(n) _____ function.

- ◯ linear
- ◯ exponential
- ◯ periodic
- ◯ odd
- ◯ NONE OF THE ABOVE

7. [10 points] We start with the function $f(x) = \cos x$ and perform the following transformations to its graph:

(i) Stretch it vertically by a factor of 2.5

(ii) Compress it horizontally by a factor of $\frac{1}{3}$

(iii) Shift it vertically, down by 1

(iv) Shift it horizontally right by π.

a. [4 points] Call the function represented by the new graph $g(x)$. What is a formula for this new function $g(x)$?

$g(x) =$ _____

b. [2 points] What is an equation for the midline of $g(x)$?

$y =$ _____

c. [2 points] What is the amplitude of $g(x)$?

Amplitude: _____

d. [2 points] What is the period of $g(x)$?

Period: _____

8. [5 points] The *I-Love-Functions Dog Hotel* has a one-of-a-kind Doggie Ferris Wheel for its residents to use on special occasions. The hotel residents board the Doggie Ferris Wheel at its lowest point, from a platform that is 5 feet high. The Doggie Ferris Wheel is 34 feet in diameter.

a. [3 points] If each full rotation rotation takes 1 minute, how high off of the ground is a dog when she is exactly 20 seconds into the ride?
Show all work (including any pictures). Give your final answer in decimal form, NOT exact form.

Height: _____ feet

b. [2 points] What length of the Doggie Ferris Wheel's arc is traversed by a passenger dog in 47 seconds of riding?
Show all work (including any pictures). Give your final answer in decimal form, NOT exact form.

Length of arc: _____ feet