# Math 105 — First Midterm — February 19, 2024

Write your 8-digit UMID number
very clearly in the box to the right,
and fill out the information on the lines below

Your Initials Only:	Your 8-digit UMID	number (not uniquame):	
0	0		

Instructor Name: \_\_\_\_\_

Section #: \_\_\_\_\_

## 1. Do not open this exam until you are told to do so.

## 2. Do not write your name anywhere on this exam.

- 3. This exam has 8 pages including this cover. There are 7 problems. Note that the problems are not of equal difficulty, so you may want to skip over and return to a problem on which you are stuck.
- 4. Do not separate the pages of this exam. If they do become separated, write your UMID (not name) on every page and point this out to your instructor when you hand in the exam.
- 5. The back of every page of the exam is blank, and, if needed, you may use this space for scratch-work. Clearly identify any of this work that you would like to have graded.
- 6. Read the instructions for each individual problem carefully. One of the skills being tested on this exam is your ability to interpret mathematical questions, so while you may ask for clarification if needed, instructors are generally unable to answer such questions during the exam.
- 7. Show an appropriate amount of work for each problem, so that graders can see not only your answer but how you obtained it.
- 8. You must use the methods learned in this course to solve all problems.
- 9. You are allowed notes written on two sides of a  $3'' \times 5''$  note card and one scientific calculator that does not have graphing or internet capabilities.
- 10. Include units in your answer where that is appropriate.
- 11. Problems may ask for answers in *exact form* or in *decimal form*. Recall that  $\sqrt{2} + \cos(3)$  is in exact form and 0.424 would be the same answer expressed in decimal form.
- 12. **Turn off all cell phones, smartphones, and other electronic devices**, and remove all headphones, earbuds, and smartwatches. Put all of these items away. The use of any networked device while working on this exam is <u>not</u> permitted.

Problem	Points	Score
1	12	
2	9	
3	6	
4	8	

Problem	Points	Score
5	6	
6	11	
7	8	
Total	60	

- **1**. [12 points] Consider the following functions:
  - f(x) = 2(x-1) 5
  - g(x) is given by the graph below.
  - Some values of the function h(x), which has an inverse function  $h^{-1}$ , are given in the table below.



x	-2	-1	0	1	2
h(x)	5	2	-1	-10	0

**a**. [2 points] Over which of the following intervals does g(x) appear to be concave up on the **entire interval**? Circle all that apply.

(-5, -3] (-3, 0) (1, 4] (2, 4] NONE

**b.** [2 points] Over which of the following intervals does g(x) appear to be increasing on the **entire interval**? Circle all that apply.

(-5, -3] (-3, 1) (2, 4] (3, 4] NONE

c. [2 points] Give a formula for a linear function w(x) whose graph is perpendicular to the graph of f(x) and goes through the point (3, -2).

w(x) = \_\_\_\_\_

- d. [6 points] Find the value of the following quantities, where possible. Write N/A if they cannot be determined or do not exist.
  - (i)  $f^{-1}(9) =$  \_\_\_\_\_
  - (ii) f(g(-3)) = \_\_\_\_\_
- (iii)  $h^{-1}(g(3)) =$  \_\_\_\_\_\_
- (iv) If w(x) = g(x-1) 3, w(2) = \_\_\_\_\_
- (v) All x such that g(x) = 1: x = \_\_\_\_\_

2. [9 points] The height of water in a cylindrical tank, as it drains out, is given by

$$H = h(t) = 4t^2 - 40t + 100,$$

where H is measured in centimeters and t is measured in minutes after a spigot is opened. The formula holds until the tank is emptied, after which, the height does not change anymore.

For your reference, the zeros of  $y = ax^2 + bx + c$  can be found by the formula:  $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ .

**a**. [2 points] How high is the water in the tank when the spigot is first opened? Give your answer in exact form, or rounded to two decimal places. Include units.

Water height: \_\_\_\_\_

**b**. [2 points] After how many minutes is the tank empty? Show all work. Give your final answer in exact form or rounded to two decimals places.

\_\_\_\_\_ minutes

**c**. [2 points] What is a reasonable domain and range for this function in the context of the problem? Use inequality OR interval notation for your answer.

Domain:

Range:

**d**. [3 points] How long does it take for the tank to be half as full as it started? Show all work. Give your final answer rounded to two decimals places.

minutes

- **3.** [6 points] On the axes below, sketch the graph of a single function y = f(x) with all of the following properties:
  - f(x) has its vertical intercept at y = 4,
  - the average rate of change of f between x = -2 and x = 0 is 0,
  - f(x) is concave down and increasing on the interval -4 < x < -1,
  - f(x) has a constant rate of change on 0 < x < 2 with slope  $-\frac{5}{2}$ ,
  - the domain of f(x) is  $-5 < x \le 5$ ,
  - the range of f(x) is  $-4 \le f(x) \le 5$ .



- 4. [8 points] For a certain computer, P(f) measures the amount of power the computer consumes in Watts (W) as a **linear** function of the frequency, f, which is measured in Gigahertz (GHz). At the frequency f = 0.8 GHz the computer consumes 8 W of power; at the frequency f = 3.4 GHz the computer consumes 125 W of power.
  - **a**. [3 points] Find the slope of P(f) and give its units. Show all work. Give your answer rounded to at least two decimal places.

**b.** [2 points] Suppose we open a new application on the computer and the frequency increases by 2.8 GHz. By how many watts (W) did the power consumption, P(f), increase? Show all work. Give your answer in exact form or rounded to at least two decimal places.

Watts

c. [3 points] Find a formula for P(f). Show all work. Express all constants in exact form or rounded to at least two decimal places.

5. [6 points] Let W(d) be the probability that the great soccer player Pelénomial scores when he takes a shot d yards away from the goal line. Some values of W(d) are given in the table below.

d	0	6	12	18
W(d)	0.94	0.4831	0.2483	

**a**. [2 points] Is W(d) modeled better by a linear function or by an exponential function? To receive credit, you must test <u>both</u> models <u>and</u> show all work.

(Circle one)

LINEAR

#### EXPONENTIAL

b. [2 points] If you said above that W(d) was linear, find its slope. If you said above that W(d) was exponential, find its approximate growth factor. Show all work or point to relevant work above. Give your answer rounded to two decimal places.

SLOPE (if linear) / GROWTH FACTOR (if exponential):

c. [2 points] Use your work above to compute the probability that Pelénomial scores when he takes a shot 18 yards away from the goal line. Show all work. Give your answer in exact form, or rounded to two decimal places.

Answer: \_\_\_\_\_

- **6**. [11 points] The following problem parts are not related.
  - **a**. [2 points] A ball is thrown up in the air from a platform and its height in meters above the ground is

$$H(t) = -4.9(t - 0.9)^2 + 4.5,$$

where t is measured in seconds. What is the greatest height above the ground the ball reaches? And when does it reach that height?

b. [4 points] Write a formula for a population of bacteria P(t) that starts with a population of  $10^5$  and grows by 30% every day. The variable t is measured in days after the experiment starts.

If E(p) is the rate at which is energy is given off, measured in joules/second, by p bacteria of this kind, what is the meaning of the following equation?

$$E(P(2)) = 0.3$$

## Meaning:

c. [5 points] A table of some values of the function h(r) is given below:

r	-2	0	2	4
h(r)	-3	-1	10	5

Let g(r) = h(r-1) + 3.

To obtain the graph of g(r), one must shift the graph of h(r)...

- ...vertically (CIRCLE ONE) UP DOWN by \_\_\_\_\_
- ...horizontally (CIRCLE ONE) LEFT RIGHT by \_\_\_\_\_

From the given information, we can deduce the coordinates of several points on the graph of g(r). Give the coordinates of two such points:

Greatest height: \_\_\_\_\_ meters

Tir

 $P(t) = _____$ 

Time: \_\_\_\_\_\_ seconds

- 7. [8 points] Mx. Miserable's Maids (or, MMM) charges
  - an \$18 "drive up" fee per house cleaning;
  - \$40 per 1000 square feet for the first 2500 square feet;
  - then \$32 per 1000 square feet beyond 2500.

For example, the cost to clean a 750 square foot house is \$18 + \$30 = \$48. Additionally, they will not take on any clients with houses greater than 5000 square feet.

**a**. [2 points] How much does MMM charge for cleaning a 3000 square foot house? Show all work. Given your answer rounded to at least two decimal places.

\_\_\_\_\_ dollars

**b.** [6 points] Let C(a) be the price, in dollars, that MMM charges for a single visit to a house that is a **thousand square feet**. Find a piecewise-defined formula for C(a):

$$C(a) = \begin{cases} & & & \\ & & & \\ & & & \\ & & & \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & \\ & & & & \\ & &$$