Math 105 — Second Midterm — April 1, 2024

Write your 8-digit UMID number very clearly in the box to the right, and fill out the information on the lines below.

Your Initials Only: _____ Your 8-digit UMID number (not uniqname): _____

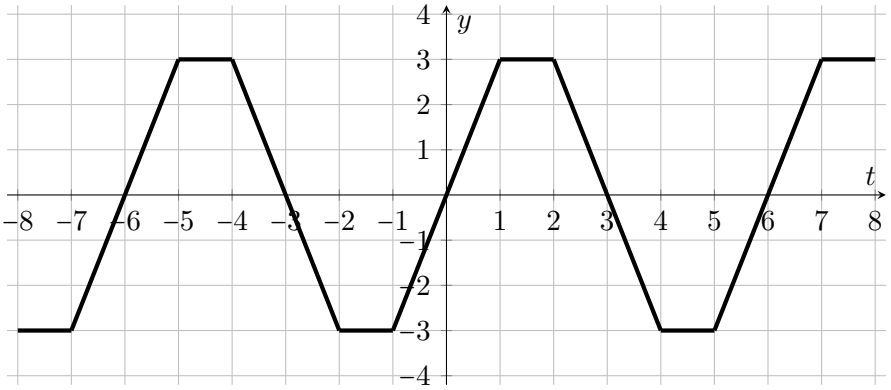
Instructor Name: _____ Section #: _____

1. **Do not open this exam until you are told to do so.**
2. **Do not write your name anywhere on this exam.**
3. This exam has 9 pages including this cover. There are 6 problems. Note that the problems are not of equal difficulty, so you may want to skip over and return to a problem on which you are stuck.
4. Do not separate the pages of this exam. If they do become separated, write your UMID (not name) on every page and point this out to your instructor when you hand in the exam.
5. The back of every page of the exam is blank, and, if needed, you may use this space for scratch-work. Clearly identify any of this work that you would like to have graded.
6. Read the instructions for each individual problem carefully. One of the skills being tested on this exam is your ability to interpret mathematical questions, so while you may ask for clarification if needed, instructors are generally unable to answer such questions during the exam.
7. Show an appropriate amount of work for each problem, so that graders can see not only your answer but how you obtained it.
8. You must use the methods learned in this course to solve all problems.
9. You are allowed notes written on two sides of a $3'' \times 5''$ note card and one scientific calculator that does not have graphing or internet capabilities.
10. Include units in your answer where that is appropriate.
11. Problems may ask for answers in *exact form* or in *decimal form*. Recall that $\sqrt{2} + \cos(3)$ is in exact form and 0.424 would be the same answer expressed in decimal form.
12. **Turn off all cell phones, smartphones, and other electronic devices**, and remove all headphones, earbuds, and smartwatches. Put all of these items away. The use of any networked device while working on this exam is <u>not</u> permitted.

Problem	Points	Score
1	12	
2	9	
3	11	

Problem	Points	Score
4	8	
5	10	
6	10	
Total	60	

1. [12 points] Below is a graph of periodic, odd function $h(t)$, with period 6:



a. [3 points] A **new** function $g(t) = -h(2t)$ ...

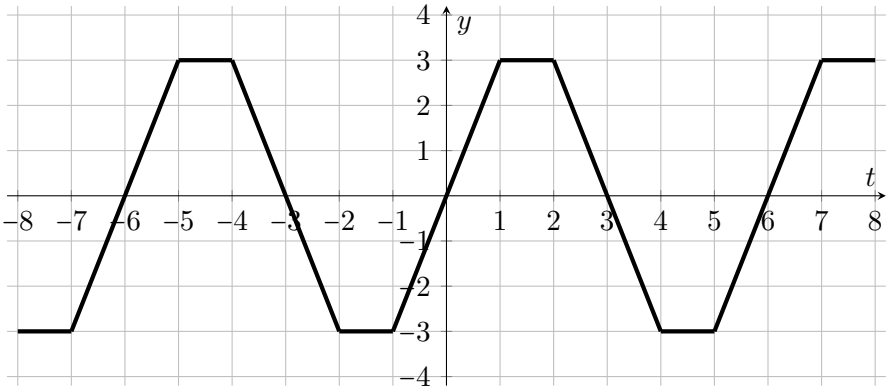
- ... has period: _____
- ... is (CIRCLE ONE) **ODD** **EVEN** **NEITHER**
- ... has amplitude: _____

b. [3 points] Another **new** function $w(t) = 3h(t-1)$ ...

- ... has period: _____
- ... is (CIRCLE ONE) **ODD** **EVEN** **NEITHER**
- ... has maximum value: _____

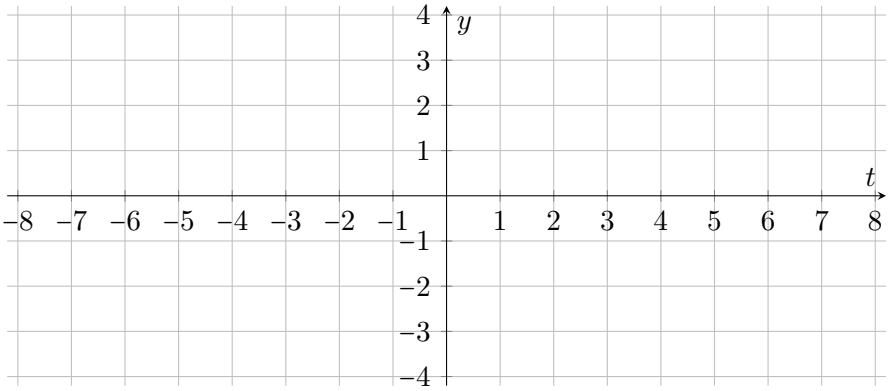
This problem continues on the next page.

The graph of $h(t)$ is reproduced here for your convenience.

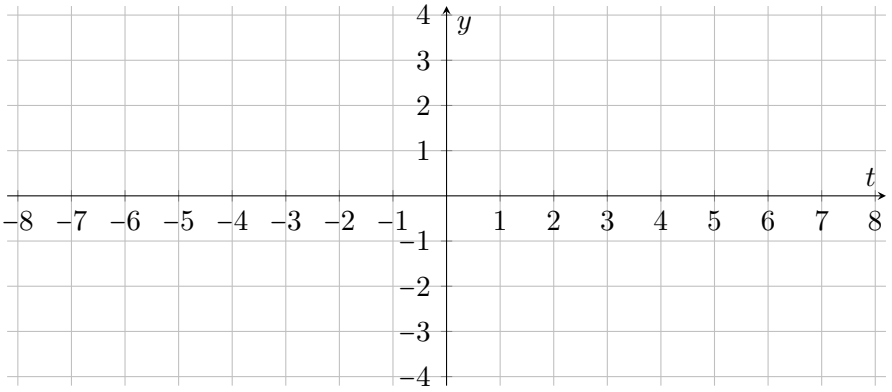


c. [6 points] Carry out the following sequence of transformations to the graph of $h(t)$. Draw each intermediate graph on the provided axes. *Clearly label at least three specific, known points in each graph.*

1. Shift the graph of $h(t)$ up by 1 unit.



2. Reflect the resulting graph over the t-axis.



Call the function in the final graph $k(t)$. What is a formula for $k(t)$ in terms of $h(t)$?

$k(t) =$ _____

2. [9 points] A group of scientists is modeling the transmission of light through different liquids. The functions below measure the brightness of the light, in lumens, at a depth of d cm below the surface of two different liquids: A and B.

$$A(d) = 45e^{-.001d}$$
$$B(d) = 50e^{-0.001(2d-25)}$$

The functions $A(d)$ and $B(d)$ have a domain of $[0, \infty)$.

a. [1 point] How bright is the light at the surface of liquid B? *Express your answer in exact form, or rounded to at least two decimal places.*

_____ lumens

b. [4 points] At what depth do the lights in the experiments with liquids A and B have the same brightness? *Show all work. Express your answer in exact form, or rounded to at least two decimal places.*

_____ cm

c. [4 points] In a third experiment the scientists observe that the brightness of a light decreases by 10% for every 5 cm of depth below the surface of a liquid C. No matter the starting depth, how much deeper do you need to go to reduce the brightness by 25%? *Show all work. Express your answer in exact form, or rounded to at least two decimal places.*

_____ cm

3. [11 points] Asteroid Mining Co. hauls mineral-rich asteroids from across the solar system back to Earth's orbit for mining. The scientists at the company use the following functions to compute instructions for the space ship's crew.

- $S(p)$ is the amount of fuel, measured in liters, that is needed to move the ship p parsecs (a unit of distance).
- $F(s)$ is the amount of fuel, measured in liters, that is used when the engines run for s seconds.

Assume that $S(p)$ and $F(s)$ are both invertible.

a. [6 points] For each of the expressions below, give an interpretation or explain why the expression doesn't make sense.

- $S^{-1}(2.3 \times 10^8) = 0.2$

- $S(F(60))$

- $F^{-1}(S(20))$

b. [2 points] Write an expression for the amount of fuel used, in liters, when the ship travels 2.8 parsecs and then runs the engines for an additional 30 seconds. Your answer may include any of S, F, S^{-1}, or F^{-1}.

_____ liters

c. [3 points] Define the function $D(m)$ to be the distance, in parsecs, the ship moves when the engines run for m **minutes**. Write a formula for $D(m)$ in terms of F, S, F^{-1}, and/or S^{-1}.

$D(m) =$ _____

4. [8 points] The following parts are unrelated.

a. [4 points] The quantity of an intravenous drug in a patient's body, in mg, is given by $D(t) = 250(0.88)^t$, where t is the number of hours after the drug was administered.

What is the hourly decay rate of the drug? *Express in exact form, or rounded to at least 2 decimal places.*

_____

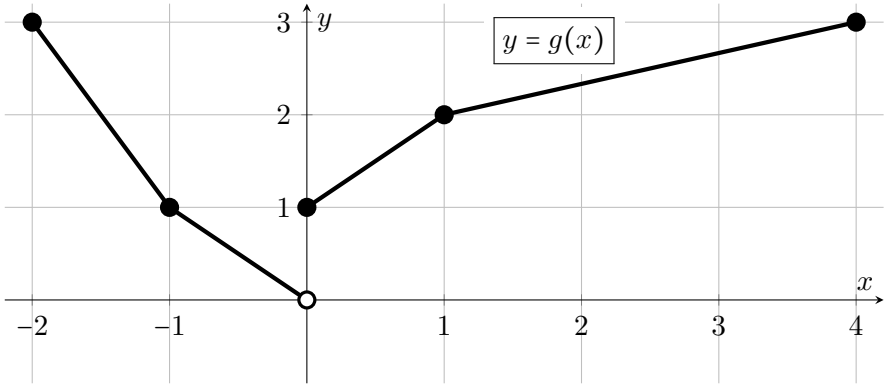
What is the ***continuous*** hourly decay rate of the drug? *Express in exact form, or rounded to at least 2 decimal places.*

_____

b. [4 points] The function $f(x)$ is given by the following formula:

$$f(x) = \ln(x) + 2$$

The entirety of the function $g(x)$ is given by the graph below.



Find the following values, or write NEI if there is "not enough information" to compute them. *Show all work.*

- $f^{-1}(g(0)) =$ _____

- All x such that $g(g(x)) = 2$: $x =$ _____

5. [10 points] The temperature T in a given room, measured in °F, after an air conditioner is turned on is given by $T = f(t) = 68 + 5e^{-0.02t}$, where t is measured in minutes.

a. [4 points] Find the following limits of $f(t)$:

(i) $\lim_{t\to\infty} f(t) =$ _____

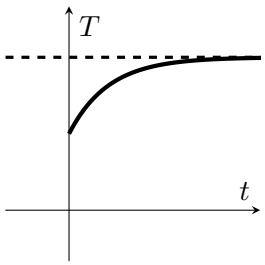
(ii) $\lim_{t\to-\infty} f(t) =$ _____

b. [3 points] Find a formula for $t = f^{-1}(T)$.

$f^{-1}(T) =$ _____

c. [3 points]

The graph of $T = q(t)$ to the right shows the temperature in a different room when being *heated* as a function of time t. The domain shown is $[0, \infty)$ and the dashed line represents a horizontal asymptote of $q(t)$.



Given that behavior, which of the following *could* be a formula for $q(t)$? (*Circle all that apply.*)

$q(t) = 3\log(t+2)$

$q(t) = 50 \cdot 1.02^t$

$q(t) = -0.7^t + 65$

$q(t) = -3\log(t+5)$

$q(t) = -e^{-0.2t} + 67$

$q(t) = -e^{0.1t} + 69$

6. [10 points] For parts (a)–(d), indicate if each of the following statements is true, false, or if there is not enough information, by circling the correct answer. **Provide a *brief* explanation of your answer.**

a. [2 points] If the function $f(x)$ is odd, then the function $q(x) = (f(x))^2$ is even.

TRUE FALSE NOT ENOUGH INFORMATION

Explanation:

b. [2 points] The function $\log(x)$ **can't** take negative numbers as inputs, but it **can** have negative numbers as outputs.

TRUE FALSE NOT ENOUGH INFORMATION

Explanation:

c. [2 points] The function $f(x) = \log(x - h) + k$, where h, k are some constants, has a vertical asymptote at $x = h$.

TRUE FALSE NOT ENOUGH INFORMATION

Explanation:

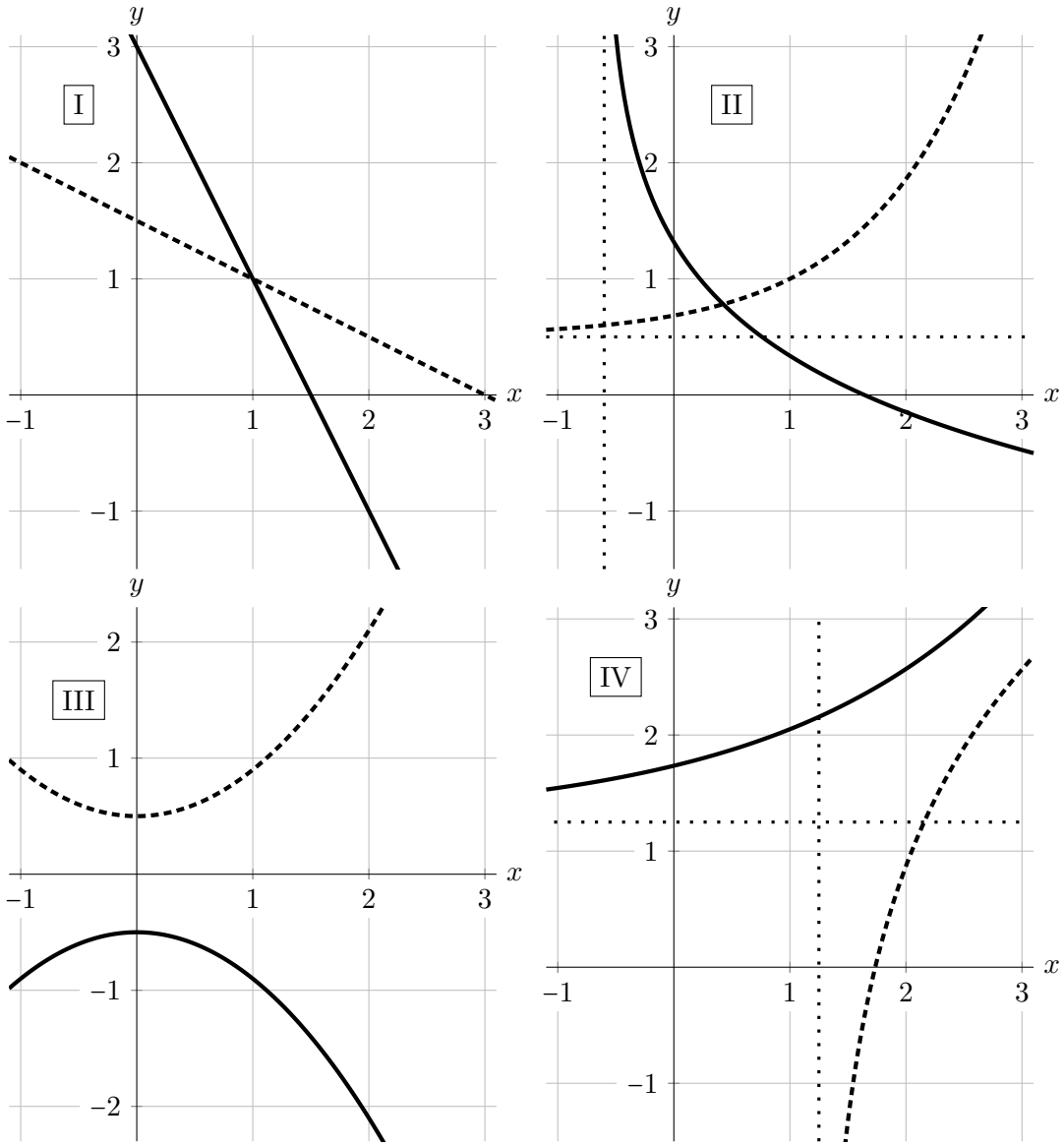
d. [2 points] If $Q(t)$ is an exponentially growing function, then the time it takes for the quantity to double gets shorter and shorter as time goes on.

TRUE FALSE NOT ENOUGH INFORMATION

Explanation:

This problem continues on the next page.

e. [2 points] On each set of axes below, a solid function and a dashed function are plotted. Dotted lines represent vertical or horizontal asymptotes.



For which pairs of functions shown is the solid function the inverse of the dashed function? *Circle all that apply. No justification required.*

I II III IV