Math 105 — Final Exam — April 25, 2024

Write your 8-digit UMID number very clearly in the box to the right, and fill out the information on the lines below.

Your Initials Only: ______ Your 8-digit UMID number (not uniqname): ____________________

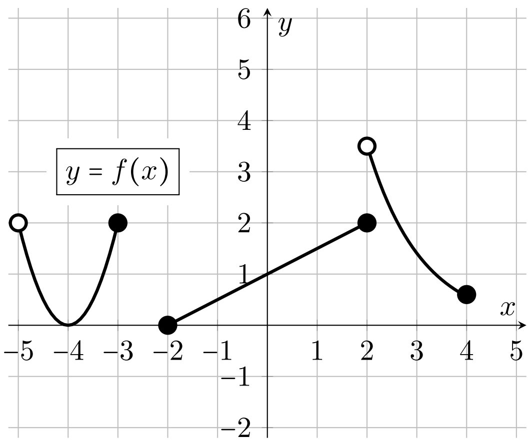
Instructor Name: ______________________________________ Section #: __________

1. **Do not open this exam until you are told to do so.**
2. **Do not write your name anywhere on this exam.**
3. This exam has 10 pages including this cover. There are 9 problems. Note that the problems are not of equal difficulty, so you may want to skip over and return to a problem on which you are stuck.
4. Do not separate the pages of this exam. If they do become separated, write your UMID (not name) on every page and point this out to your instructor when you hand in the exam.
5. The back of every page of the exam is blank, and, if needed, you may use this space for scratch-work. Clearly identify any of this work that you would like to have graded.
6. Read the instructions for each individual problem carefully. One of the skills being tested on this exam is your ability to interpret mathematical questions, so while you may ask for clarification if needed, instructors are generally unable to answer such questions during the exam.
7. Show an appropriate amount of work for each problem, so that graders can see not only your answer but how you obtained it.
8. You must use the methods learned in this course to solve all problems.
9. You are allowed notes written on two sides of a $3'' \times 5''$ note card and one scientific calculator that does not have graphing or internet capabilities.
10. Include units in your answer where that is appropriate.
11. Problems may ask for answers in *exact form* or in *decimal form*. Recall that $\sqrt{2} + \cos(3)$ is in exact form and 0.424 would be the same answer expressed in decimal form.
12. **Turn off all cell phones, smartphones, and other electronic devices**, and remove all headphones, earbuds, and smartwatches. Put all of these items away. The use of any networked device while working on this exam is <u>not</u> permitted.

Problem	Points	Score
1	9	
2	9	
3	8	
4	11	
5	6	

Problem	Points	Score
6	8	
7	10	
8	11	
9	8	
Total	80	

1. [9 points] The entire graph of a function $f(x)$ is shown below to the left. Also shown is a table of some values for an invertible function $g(x)$, and formula for a function $h(x)$.



x	-3	-2	-0.5	0	1	2
$g(x)$	6	-5	0	4	7	9

$$h(x) = \begin{cases} \dfrac{1}{x}, & -\infty < x < 0 \\ \cos(\pi x), & 0 \le x < \infty \end{cases}$$

a. [2 points] Find the **domain** of $f(x)$. Give your answers using interval notation or using inequalities. *You do not need to explain or justify your answer.*

Domain: ______________________________

b. [2 points] Find the **range** of $h(x)$ (the function given by a **formula**). Give your answers using interval notation or using inequalities. *Show all work, including any computations or graph sketches.*

Range: ______________________________

c. [5 points] Find the value of each of the following; write N/A if a value does not exist or there is not enough information to find it. *Showing work is not required, but may make you eligible for partial credit in some cases.*

(i) $g(f(2)) =$ ______________________________

(ii) $h(g^{-1}(0)) =$ ______________________________

(iii) All x such that $h(x) = -5$, $x =$ ______________________________

(iv) $g(h(2)) =$ ______________________________

(v) If $q(x) = \frac{2}{3}f(x-2)$, then $q(-1) =$ ______________________________

2. [9 points] The parts of this problem are unrelated.

a. [6 points] Consider the quadratic function $f(x) = (x-1)^2 - 1$.

(i) Find the zero(s) and vertex of $f(x)$. *Show any relevant work.*

vertex: ____________ zero(s): $x =$ ____________

(ii) Find the vertex of $f(2x) + 1$.

vertex: ____________

(iii) Find the zero(s) of $3f(x-1)$.

zero(s): $x =$ ____________

b. [3 points] A different quadratic function, $g(x)$, has its vertex at $(2, 3)$ and a zero at $x = -1$.

(i) What is the x-coordinate of the other zero of $g(x)$? (CIRCLE ONE)

0 1 2 3 4 5 NONE OF THESE NOT ENOUGH INFORMATION

(ii) What is the sign of the leading coefficient of $g(x)$? (CIRCLE ONE AND EXPLAIN)

negative positive zero NOT ENOUGH INFORMATION

Explanation of (ii):

3. [8 points] A local clothing store *Amaizing T-Shirts* sets the price per t-shirt based on how many t-shirts a customer purchases. Let $P(s)$ be the **price per t-shirt**, in dollars, when a customer purchases s t-shirts. Note that $P(s)$ is an invertible function.

a. [2 points] Describe the meaning of $P^{-1}(10)$ in the context of the problem.

b. [2 points] Write expression for the **total price** for 156 t-shirts. Your answer may involve P and/or P^{-1}.

Answer: \$ ____________________

c. [4 points] *Amaizing T-Shirts* offers a customer loyalty program. When a customer buys s t-shirts, they get $L(s)$ loyalty points. Describe the meaning of the following equations or explain why they don't make sense in context:

$L(100) = 10$

$P(L^{-1}(20)) = 9$

4. [11 points] As stated in the problem above, the price of each t-shirt $P(s)$ (in dollars) at *Amaizing T-Shirts* is a function of s, the total number of t-shirts a customer orders. In particular, now assume that if the customer orders exactly 1 t-shirt, it costs \$14.50. If the customer orders 30 t-shirts, each shirt costs \$13.30.

a. [3 points] If we assume that $P(s)$ is a **linear function**, find a formula for $P(s)$ *Show all work. Numbers in your final function can be rounded to two decimal places or expressed in exact form.*

$P(s) =$ ________________________

b. [2 points] What is the meaning of the slope of $P(s)$ in your linear function above?

Meaning of Slope:

c. [3 points] If we assume that $P(s)$ is an **exponential function**, find a formula for $P(s)$ *Show all work. Numbers in your final function can be rounded to two decimal places or expressed in exact form.*

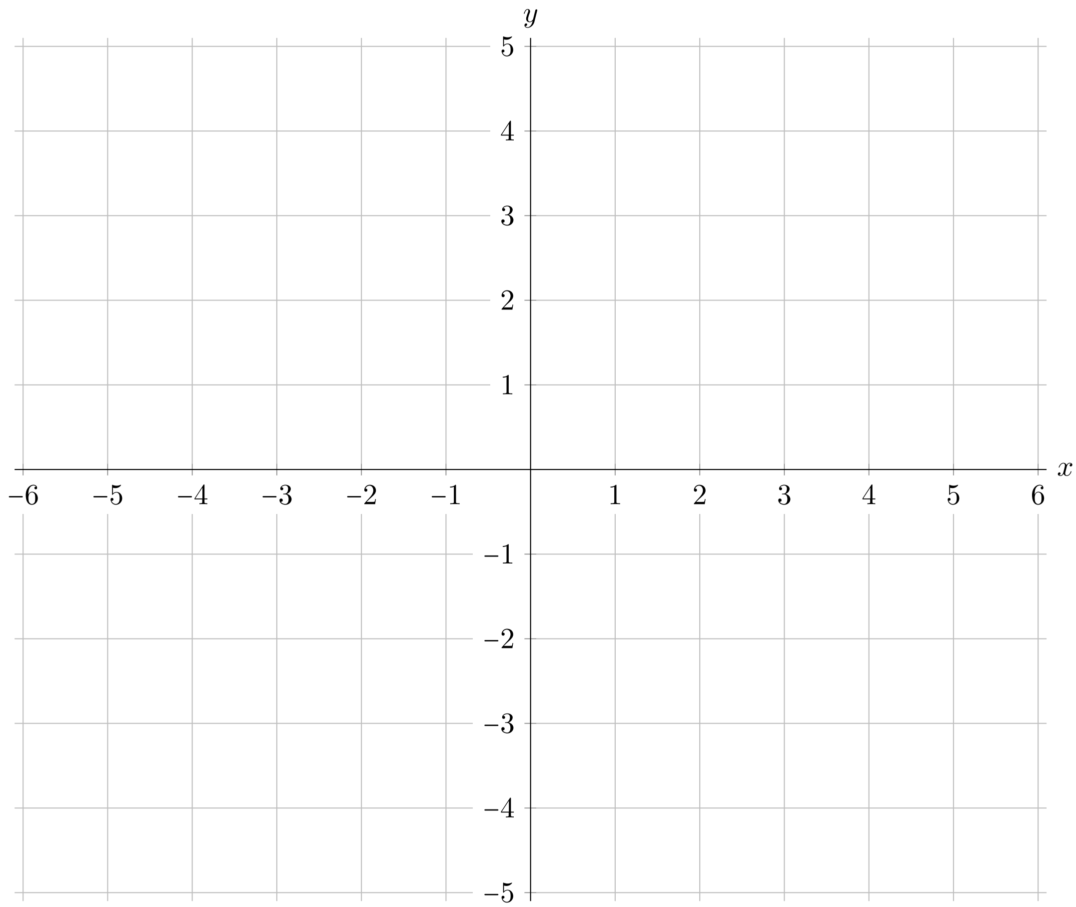
$P(s) =$ ________________________

d. [3 points] If we assume that $P(s)$ is a **power function**, find a formula for $P(s)$ *Show all work. Numbers in your final function can be rounded to two decimal places or expressed in exact form.*

$P(s) =$ ________________________

5. [6 points] On the axes below, sketch the graph of a **single** function $y = f(x)$ with all of the following properties:

- $f(2) = 4$
- $f(x)$ has domain $[-6, 6]$.
- $f(x)$ is an odd function.
- $f(x)$ is concave down **and** increasing on the domain $(0, 2)$.
- $f(x)$ is linear with constant slope $-\frac{1}{2}$ on domain $[2, 6]$.



6. [8 points] For each of the statements below, circle TRUE or FALSE and *briefly explain* your reasoning. *Credit will only be given for a reasonable explanation—circling alone is no credit.*

a. [2 points] A degree-5 polynomial will always have 5 zeros.

TRUE FALSE NOT ENOUGH INFORMATION

Explanation:

b. [2 points] Because a right triangle cannot contain any angles greater than $90°$, we cannot find the sine or cosine of any angles greater than $90°$.

TRUE FALSE NOT ENOUGH INFORMATION

Explanation:

c. [2 points] If $f(x)$ is an even function with domain $[-4, 4]$, then $f(x)$ is *definitely not* invertible.

TRUE FALSE NOT ENOUGH INFORMATION

Explanation:

d. [2 points] The function $w(r) = 0.4e^{0.15r}$ is an exponentially decreasing function of r.

TRUE FALSE NOT ENOUGH INFORMATION

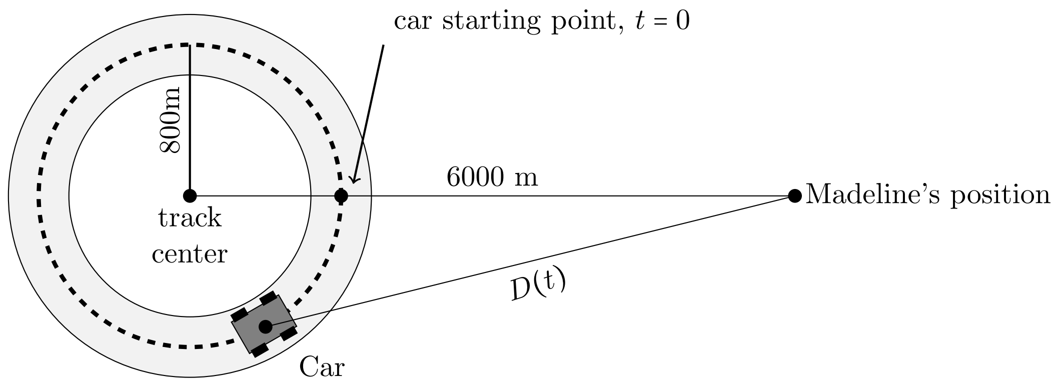
Explanation:

7. [10 points] Madeline is the head engineer of a new racetrack called the "Michigan Raceway". Unusually, this track will be a perfect circle. The radius a car's path will be 800m.

a. [2 points] While driving the test car, Madeline drives at a perfectly constant speed and makes one complete revolution in 1.5 minutes. How far, in meters, does Madeline drive in 40 seconds? *Show all work. Give your answer in exact form, or rounded to two decimal places.*

____________________ meters

b. [4 points] While another engineer takes over the test car, Madeline stands directly east, 6000m away from **center** of the track. The car now drives at a constant speed and takes exactly 2 minutes for each lap. Let $D(t)$ be the test car's distance from Madeline, in meters, where t is measured in minutes since the car started, directly east of center.



(i) What is the minimum value of $D(t)$? ______________ meters

(ii) What is the maximum value of $D(t)$? ______________ meters

(iii) What is the exact value of $D(0.5)$? *Show all work.*

______________ meters

c. [4 points] If we want to *approximate* a formula for $D(t)$ using a sinusoidal function of the form $D(t) = A\cos(B(t-h)) + k$, what formula should we use? That is, use the information above to find the appropriate values for A, B, h, and k and write out the final formula.

$D(t) =$ ____________________________________

8. [11 points]

a. [6 points] Find the following limits. They will either be a real number, ∞, or $-\infty$.
You don't need to show work but partial credit may be awarded for work shown.

(i)

$$\lim_{x\to-\infty} \frac{x^3+x^2+100}{3x^2-x} = \_\_\_\_\_\_\_\_\_\_$$

(ii)

$$\lim_{x\to\infty} \frac{(7x-2)^2(x+3)}{5x^3+1} = \_\_\_\_\_\_\_\_\_\_$$

(iii)

$$\lim_{x\to\infty} \frac{e^x-x^4}{-(10^x)+x^2} = \_\_\_\_\_\_\_\_\_\_$$

b. [5 points] Consider the rational function:

$$f(x) = \frac{3x^2(x-3)}{(x+4)(x-1)^2(x-3)}$$

Find the following features or write NONE is none exist. *Show all relevant work.*

(i) Coordinates of any hole(s): ____________________

(ii) Equations for any horizontal asymptote(s): ____________________

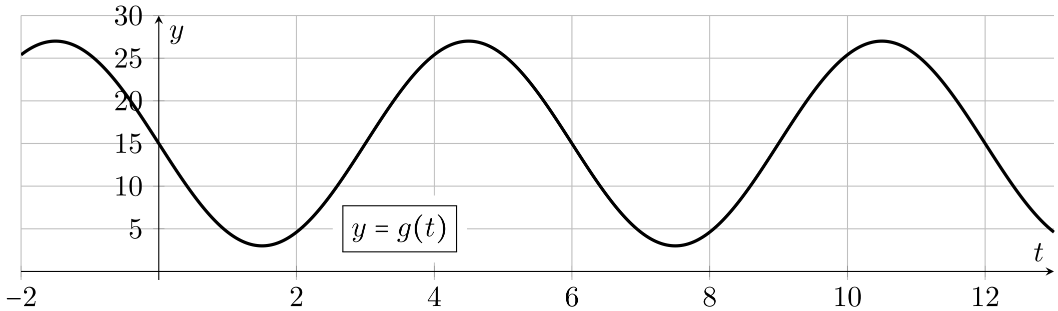
(iii) Equations for any vertical asymptotes(s): ____________________

9. [8 points] The following problem parts are not related.

a. [3 points] The function $g(t)$, shown in the graph below, is a sinusoidal function with

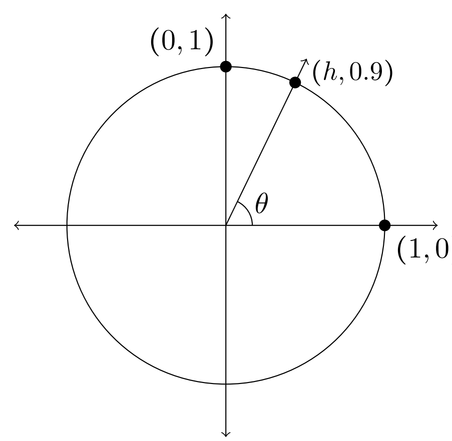
- period 6
- midline $y = 15$
- and y-intercept $(0, 15)$

Using the fact that $g(-0.41) = 20$, find all other solutions to $g(t) = 20$ on the domain $[0, 12]$ and illustrate on the graph where they fall using dots.



$t =$ ________________________

b. [5 points] In the unit circle shown below, the ray at angle θ to the positive x-axis intersects the unit circle at the coordinates $(h, 0.9)$.



(i) What is the value of h? *Show all relevant work. Give your final answer in exact form, or accurate to two decimal places.*

$h =$ __________

(ii) What is the value of θ in degrees? *Show all relevant work. Give your final answer in* ***numerical*** *form, accurate to two decimal places.*

$\theta =$ __________ $^\circ$

(iii) Find the value for an angle ϕ (in degrees), between 90° and 180°, such that $\sin(\phi) = 0.9$. *Show all relevant work. Give your final answer in terms of θ or as a number rounded to two decimal places.*

$\phi =$ __________ $^\circ$

(iv) Find **all** possible values of an angle ω (in degrees) between 0 and 360° such that $\cos(\omega) = 0.9$. *Show all relevant work. Give your final answer in terms of θ or as a number rounded to two decimal places.*

$\omega =$ ________________________ $^\circ$