1. [20 points] Use the functions $g$, $h$, $p$, and $k$ given below to answer the questions that follow. 

*Note: Some answers may involve the constant $b$.*

<table>
<thead>
<tr>
<th>$t$</th>
<th>-4</th>
<th>-2</th>
<th>0</th>
<th>2</th>
<th>4</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>$g(t)$</td>
<td>4</td>
<td>$b$</td>
<td>2</td>
<td>1</td>
<td>-2</td>
<td>-b</td>
</tr>
</tbody>
</table>

$h(y) = \frac{2y}{y^2 + 1}$

$p(x) = \begin{cases} 
(x + 4)^2 - 5 & \text{for } -3 \leq x \leq -1 \\
1.2(0.2)^x & \text{for } x > -1 
\end{cases}$

**a.** [2 points] Evaluate $p(-1) + p(1)$.

**Solution:**

$$p(-1) + p(1) = ((-1 + 4)^2 - 5) + 1.2(0.2)^1 = ((3)^2 - 5)) + 1.2(0.2) = 4 + 0.24 = 4.24$$

**Answer:** 4.24

**b.** [2 points] Evaluate $p(k(0))$.

**Solution:**

$$p(k(0)) = p(2) = 1.2(0.2)^2 = 1.2(0.04) = 0.048$$

**Answer:** 0.048

**c.** [2 points] Evaluate $h(g(-2) + 2)$.

**Solution:**

$$h(g(-2) + 2) = h(b + 2) = \frac{2^{b+2}}{(b + 2)^2 + 1} = \frac{2^{b+2}}{b^2 + 4b + 5}$$

**Answer:** $\frac{2^{b+2}}{b^2 + 4b + 5}$

**d.** [2 points] Solve $k(m) = b$ for $m$.

**Solution:** Since $k(2) = b$ (and no other input gives an output of $b$) we see that the unique solution is $m = 2$.

**Answer:** $m = 2$

**e.** [2 points] Assume $g$ and $k$ are invertible. Evaluate $g^{-1}(-2) + k^{-1}(0)$.

**Solution:**

$$g^{-1}(-2) + k^{-1}(0) = 4 + (-b) = 4 - b$$

**Answer:** $4 - b$

This problem continues on the next page.
This is a continuation of the problem from the previous page.

Recall that \( h(y) = \frac{2y}{y^2 + 1} \) and \( p(x) = \begin{cases} (x + 4)^2 - 5 & \text{for } -3 \leq x \leq -1 \\ 1.2(0.2)^x & \text{for } x > -1. \end{cases} \)

f. [3 points] Find the domain of \( h \). Use either inequalities or interval notation to give your answer. Please remember to show your work.

**Solution:** Since \( 2y \) and \( y^2 + 1 \) are both defined for all values of \( y \), the only possible restriction is that the denominator cannot be zero. However, \( y^2 + 1 > 0 \) for all real values of \( y \), so the domain of \( h \) is the set of all real numbers.

**Domain:** \((-\infty, \infty)\)

g. [3 points] Find the domain of \( p \). Use either inequalities or interval notation to give your answers. Please remember to show your work.

**Solution:** The first piece of the formula defines the function for values of \( x \) in the interval \([-3, -1]\). The second piece does so for values of \( x \) in the interval \((-1, \infty)\). Hence the function is defined for all values of \( x \) in the interval \([-3, \infty)\).

**Domain:** \([-3, \infty)\) (All real numbers \( x \) with \( x \geq -3 \))

h. [4 points] Find the range of \( p \). Use either inequalities or interval notation to give your answers. Please remember to show your work; this includes sketching any graphs you use.

**Solution:**

The graph of \( y = p(x) \) is shown to the right. From the graph, we see that the smallest output of the function \( p \) on its domain is \(-4\). The other outputs are all the real numbers from \(-4\) up to (but not including) \( 6 \).

**Range:** \([-4, 6)\) (All real numbers \( y \) with \(-4 \leq y < 6\))