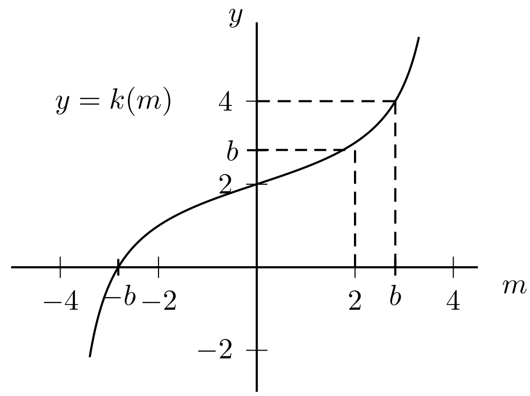
1. [20 points] Use the functions g, h, p, and k given below to answer the questions that follow. *Note: Some answers may involve the constant b.*

t	-4	-2	0	2	4	6
$g(t)$	4	b	2	1	-2	-b

$$h(y) = \frac{2^y}{y^2+1}$$

$$p(x) = \begin{cases} (x+4)^2 - 5 & \text{for } -3 \le x \le -1 \\ 1.2(0.2)^x & \text{for } x > -1 \end{cases}$$



a. [2 points]
Evaluate $p(-1) + p(1)$.

Solution:
$p(-1) + p(1) = \big((-1+4)^2 - 5\big) + 1.2(0.2)^1 = \big((3)^2 - 5)\big) + 1.2(0.2) = 4 + 0.24 = 4.24$

Answer: $\boxed{4.24}$

b. [2 points] Evaluate $p(k(0))$.

Solution: $p(k(0)) = p(2) = 1.2(0.2)^2 = 1.2(0.04) = 0.048$

Answer: $\boxed{0.048}$

c. [2 points] Evaluate $h(g(-2)+2)$.

Solution: $h(g(-2)+2) = h(b+2) = \dfrac{2^{b+2}}{(b+2)^2+1} = \dfrac{2^{b+2}}{b^2+4b+5}$

Answer: $\boxed{\dfrac{2^{b+2}}{b^2+4b+5}}$

d. [2 points] Solve $k(m) = b$ for m.

Solution: Since $k(2) = b$ (and no other input gives an output of b) we see that the unique solution is $m = 2$.

Answer: $\boxed{m = 2}$

e. [2 points] Assume g and k are invertible. Evaluate $g^{-1}(-2) + k^{-1}(0)$.

Solution: $g^{-1}(-2) + k^{-1}(0) = 4 + (-b) = 4 - b$

Answer: $\boxed{4 - b}$

This problem continues on the next page.

This is a continuation of the problem from the previous page.

Recall that $h(y) = \dfrac{2^y}{y^2+1}$ and $p(x) = \begin{cases} (x+4)^2 - 5 & \text{for } -3 \le x \le -1 \\ 1.2(0.2)^x & \text{for } x > -1. \end{cases}$

f. [3 points] Find the domain of h. Use either inequalities or interval notation to give your answer. Please remember to show your work.

Solution: Since 2^y and y^2+1 are both defined for all values of y, the only possible restriction is that the denominator cannot be zero. However, $y^2+1 > 0$ for all real values of y, so the domain of h is the set of all real numbers.

Domain: $(-\infty, \infty)$

g. [3 points] Find the domain of p. Use either inequalities or interval notation to give your answers. Please remember to show your work.

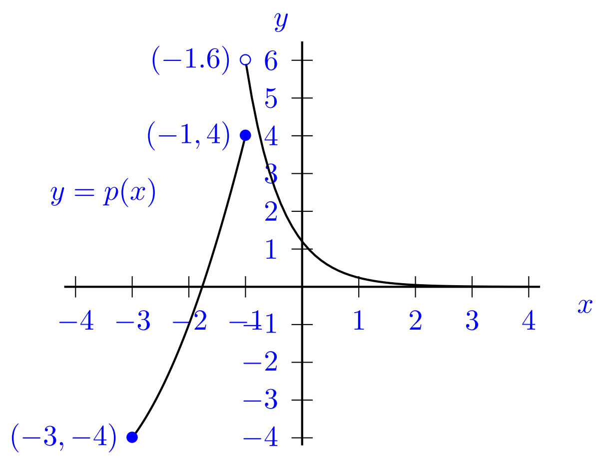
Solution: The first piece of the formula defines the function for values of x in the interval $[-3,-1]$. The second piece does so for values of x in the interval $(-1,\infty)$. Hence the function is defined for all values of x in the interval $[-3,\infty)$

Domain: $[-3,\infty)$ (All real numbers x with $x \ge -3$)

h. [4 points] Find the range of p. Use either inequalities or interval notation to give your answers. Please remember to show your work; this includes sketching any graphs you use.

Solution:

The graph of $y = p(x)$ is shown to the right. From the graph, we see that the smallest output of the function p on its domain is -4. The other outputs are all the real numbers from -4 up to (but not including) 6.



Range: $[-4, 6)$ (All real numbers y with $-4 \le y < 6$)