

4. [16 points] Marisa is planning to open a lemonade stand, and she needs to buy equipment and ingredients to make the lemonade. If she decides to make a total of 12 gallons of lemonade, the equipment and ingredients will cost her a total of 57 dollars. However, if she decides to make 20 gallons, it will cost her 85 dollars.
- a. [5 points] Let $C(g)$ be the cost to Marisa, in dollars, of producing g gallons of lemonade. Assuming $C(g)$ is a linear function, find a formula for $C(g)$.

Solution: From the information provided in the problem statement, we know $C(12) = 57$ and $C(20) = 85$.

First, we find the average rate of change (slope) of C :

$$\frac{\Delta C}{\Delta g} = \frac{C(20) - C(12)}{20 - 12} = \frac{85 - 57}{20 - 12} = \frac{28}{8} = 3.5$$

Using point-slope form, we then find that $C(g) - 57 = 3.5(g - 12)$ so $C(g) = 57 + 3.5(g - 12)$.

Answer: $C(g) = 57 + 3.5(g - 12)$ or (in slope-intercept form) $C(g) = 15 + 3.5g$

- b. [3 points] Find and give a practical interpretation, in the context of this problem, of the slope of the function $C(g)$. *Include units.*

Solution: From part (a) above the slope is 3.5 dollars per gallon.

In the context of this problem, this means that it costs Marisa an additional \$3.50 for the ingredients for each additional gallon of lemonade she decides to make.

- c. [2 points] Find the vertical intercept of the function $C(g)$. *Include units.*

Solution: The vertical intercept is $C(0) = 15$ dollars.

Answer: 15 dollars

This problem continues on the next page.

This is a continuation of the problem from the previous page. For your convenience, the original problem statement has been reprinted here.

Marisa is planning to open a lemonade stand, and she needs to buy equipment and ingredients to make the lemonade. If she decides to make a total of 12 gallons of lemonade, the equipment and ingredients will cost her a total of 57 dollars. However, if she decides to make 20 gallons, it will cost her 85 dollars.

- d. [3 points] Marisa sells lemonade for 25 cents per cup (there are 16 cups in one gallon of lemonade). Assuming she can sell all of the lemonade she makes, find a formula for $R(g)$, the total amount of money (in dollars) Marisa takes in from lemonade sales, i.e. her revenue, if she makes g gallons of lemonade.

Solution: If Marisa sells g gallons of lemonade, then she sells $16g$ cups of lemonade. At 25 cents (or 0.25 dollars) per cup, this means that her revenue (in dollars) is $R(g) = 16g(0.25) = 4g$.

Answer: $R(g) = 4g$

- e. [3 points] What is the minimum number of gallons of lemonade Marisa needs to make in order not to lose money (that is, how much lemonade does she need to make to break even)?

Solution: To break even, Marisa needs her revenue to be greater than or equal to her cost.

$$\begin{aligned}R(g) &\geq C(g) \\4g &\geq 15 + 3.5g \\0.5g &\geq 15 \\g &\geq 30\end{aligned}$$

So, in order to not lose money, Marisa must make at least 30 gallons of lemonade.