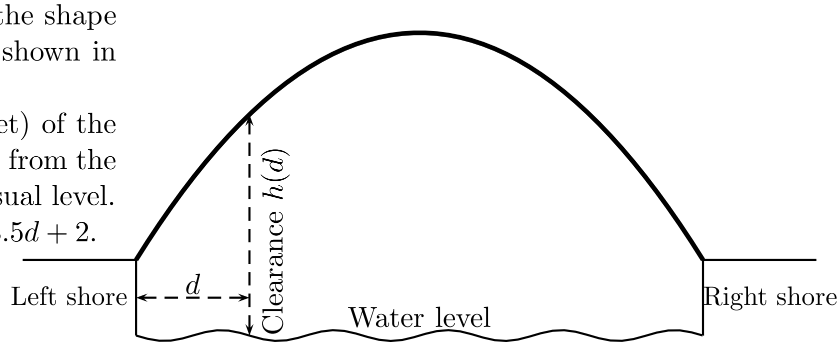
5. [16 points]

A bridge over the Huron River has the shape of a (symmetric) parabolic arch, as shown in the figure on the right.
Let $h(d)$ denote the clearance (in feet) of the bridge over a point in the river d feet from the left shore when the water is at its usual level. We are told that $h(d) = -0.07d^2 + 3.5d + 2$.



a. [8 points] By completing the square, find the maximum clearance of the bridge (that is, the clearance of the bridge at its highest point). *Remember to include units.*

Solution: We use the method of completing the square to find the vertex of the parabola that is the graph of the function $h(d)$.

$$\begin{aligned} h(d) &= -0.07d^2 + 3.5d + 2 \\ &= -0.07(d^2 - 50d) + 2 \\ &= -0.07(d^2 - 50d + (-25)^2 - (-25)^2) + 2 \\ &= -0.07((d-25)^2 - 625) + 2 \\ &= -0.07(d-25)^2 - 0.07(-625) + 2 \\ &= -0.07(d-25)^2 + 43.75 + 2 \\ &= -0.07(d-25)^2 + 45.75 \end{aligned}$$

Therefore, the vertex of the parabola is at the point $(25, 45.75)$. In terms of the bridge, the maximum clearance occurs at the vertex and is equal to 45.75 feet.

Answer: The maximum clearance of the bridge is 45.75 feet.

b. [3 points] At the bridge crossing, what is the width of the river (distance from left shore to right shore)? *Remember to include units.*

Solution: Since the bridge is symmetric, the highest point is at the point halfway across the river. From part (a), we know this point is at a distance of 25 feet from the left shore. Hence the distance from the left to the right shore is twice that amount, which is 50 feet.

Answer: The width of the river at the bridge crossing is 50 feet.

This problem continues on the next page.

This is a continuation of the problem from the previous page. For your convenience, the original problem statement has been reprinted here.

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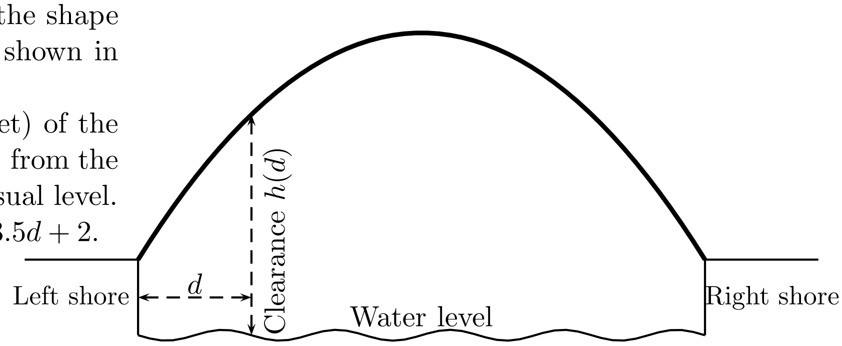

c. [5 points] Murphy is rafting down the Huron river. As Murphy's raft is passing under the bridge, he decides to pull his raft over to the left shore of the river. Murphy is six feet tall. How close to the shore can he get before he hits his head on the bridge? (Assume that Murphy is standing upright, and that the height of the raft is negligible.)
Find an answer in *exact form* and then give an approximate value accurate to at least 2 decimal places.

Solution: We want to find d so that $h(d) = 6$. So we have $-0.07d^2 + 3.5d + 2 = 6$ or $-0.07d^2 + 3.5d - 4 = 0$. Applying the quadratic formula to this equation we find

$$\begin{aligned} d &= \frac{-3.5 \pm \sqrt{(3.5)^2 - 4(-0.07)(-4)}}{2(-0.07)} \\ &= \frac{-3.5 \pm \sqrt{12.25 - 1.12}}{-0.14} \\ &= \frac{-3.5 \pm \sqrt{11.13}}{-0.14} \end{aligned}$$

Note that both of these solutions are positive. However, since we want to know how close Murphy can get, we want the smaller of these two solutions. (The other gives the point that is as close to the right shore as possible.)
Hence Murphy can get to a distance of $\dfrac{-3.5 + \sqrt{11.13}}{-0.14}$ or approximately 1.17 feet of the left shore before he hits his head on the bridge.