6. [18 points] After a particularly rainy spring and early summer, the local mosquito population grew rapidly. A local group studying the mosquito population used traps to estimate the daily mosquito population. On the 15th day of their study, 750 mosquitoes were caught in these traps, and on the 32nd day, there were 6600 mosquitoes caught in these traps.

a. [8 points] Assuming the mosquito population grew at a constant percent rate for the first 32 days of the study, find a formula for $M(t)$, the number of mosquitoes caught in the traps on day t of the study for the first 32 days of the study. Any numbers appearing in your formula should either be in *exact form* or be *accurate to at least* 4 *decimal places.*

Solution: Since the population grew at a constant percent rate, the $M(t)$ is an exponential function. Thus $M(t)$ can be written as $M(t) = ab^t$ for some a and b. We find a and b using the data provided, i.e. that $M(15) = 750$ and $M(32) = 6600$.

We have $750 = ab^{15}$ and $6600 = ab^{32}$. Taking the ratio of these quantities, we find $\dfrac{6600}{750} = \dfrac{ab^{32}}{ab^{15}}$ so $8.8 = b^{17}$ and thus $b = (8.8)^{1/17} \approx 1.13647$.

To find a we use our computed value of b in one of the earlier equations: $750 = a(8.8^{1/17})^{15}$ so $750 = a(8.8^{15/17})$ and thus $a = 750(8.8^{-15/17}) \approx 110.0764$.

Answer: $\boxed{M(t) = 750(8.8^{-15/17})(8.8^{1/17})^t \text{ or } M(t) = 110.0764(1.1365)^t}$

b. [2 points] By what percent does the number of mosquitoes caught in the traps increase each day during the first 32 days of the study?
Your answer should be accurate to at least 2 decimal places.

Solution: Using the formula we found above, the daily growth factor is approximately 1.1365 so the daily growth rate is approximately $1.1365 - 1 = 0.1365$. Hence, the number of mosquitoes caught in the traps increased by $\boxed{\text{about } 13.65\%}$ each day during the first 32 days of the study.

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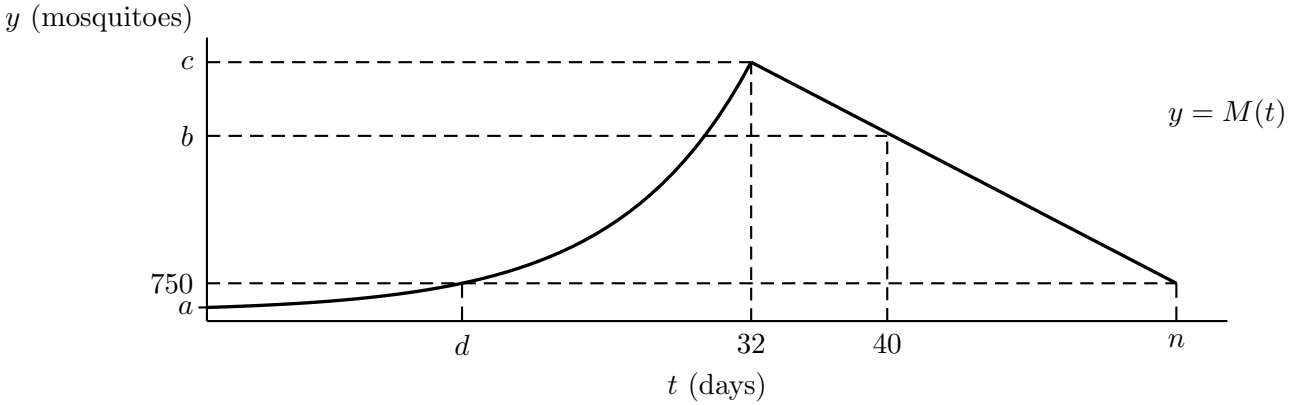
This is a continuation of the problem from the previous page.

After reaching a peak of 6600, the number of mosquitoes caught in the traps each day began to decline at a constant rate, and on the 40th day of the study, the total number of mosquitoes caught in the traps was only 5560. The constant rate of decline continued until day n, when the total number of mosquitoes collected in the traps was back down to 750.
Recall that $M(t)$ is the number of mosquitoes caught in the traps on day t of the study. A graph of $M(t)$ is shown below. *Note that the graph is not drawn to scale.*



c. [4 points] Find the values of the constants a, b, c, and d shown in the graph above.

$a =$ 110.0764 $\qquad$ $c =$ 6600

$b =$ 5560 $\qquad$ $d =$ 15

d. [4 points] Find the value of n.

Solution: Since the number of mosquitoes caught declines at a constant rate beginning on day 32, $M(t)$ is *linear* for $32 < t \leq n$.
The average rate of change of M from $t = 32$ to $t = 40$ is

$$\frac{\Delta M}{\Delta t} = \frac{M(40) - M(32)}{40 - 32} = \frac{5560 - 6600}{8} = \frac{-1040}{8} = -130 \text{ mosquitoes per day.}$$

Thus, using point-slope form, a formula for $M(t)$ for $32 < t \leq n$ is $M(t) = 6600 - 130(t - 32)$.
Since $M(n) = 750$, we solve $750 = 6600 - 130(n - 32)$ for n.

$$\begin{aligned} 750 &= 6600 - 130(n - 32) \\ -5850 &= -130(n - 32) \\ 45 &= n - 32 \\ 77 &= n \end{aligned}$$

Answer: $n =$ 77