

1. [14 points] A student named U.M. Student needs to choose a data plan for her new smartphone. She has three plans to choose from.

- The **Maize** Plan costs \$30 per month plus an additional \$10 per gigabyte of data used.
- The **Blue** Plan costs \$15 per month plus an additional \$20 per gigabyte of data used.
- The **Wolverine** Plan costs \$100 per month including unlimited data usage.

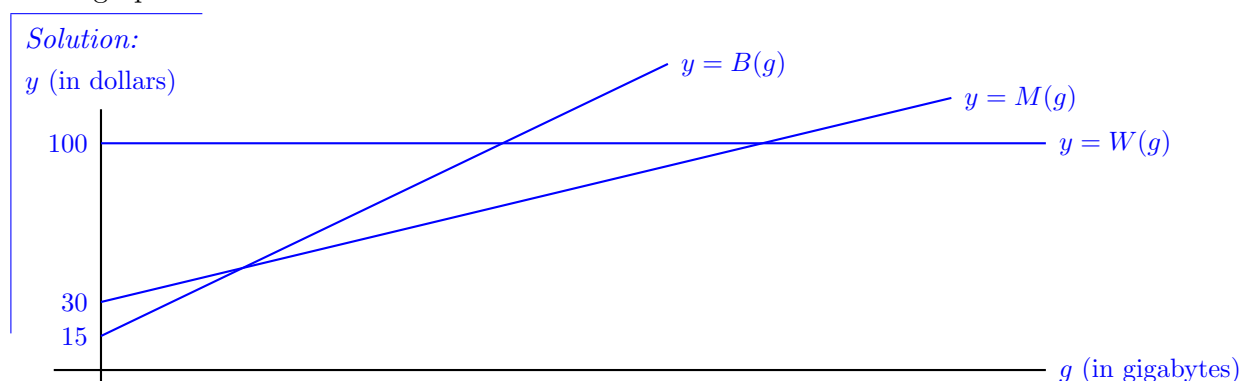
- a. [4 points] Let $M(g)$, $B(g)$, and $W(g)$ be the total cost, in dollars, for a month in which she uses g gigabytes of data under the Maize, Blue, and Wolverine Plans, respectively. Find a formula for each of these functions.

$$M(g) = \underline{\hspace{2cm} 30 + 10g \hspace{2cm}}$$

$$B(g) = \underline{\hspace{2cm} 15 + 20g \hspace{2cm}}$$

$$W(g) = \underline{\hspace{2cm} 100 \hspace{2cm}}$$

- b. [4 points] Sketch the graphs of each of these three functions on the axes below. Be sure to label the axes appropriately, including the values of any intercepts, and clearly indicate which graph is which.



- c. [2 points] If U.M. Student expects to use 1 gigabyte of data per month, which plan would be cheapest? (Justify your answer.)

Solution: $M(1) = 30 + 10(1) = 40$, $B(1) = 15 + 20(1) = 35$, and $W(1) = 100$, so the Blue Plan would be cheapest. (The price of one month of service using 1 gigabyte of data under the Maize, Blue, and Wolverine Plans are \$40, \$35, and \$100, respectively.)

- d. [4 points] Under what circumstances is the Maize Plan the cheapest? (In other words, for exactly what quantities of monthly data usage is the Maize Plan the cheapest of the three options?) *Show your work and/or explain your reasoning clearly.*

Solution: From our graph above, we see that the Maize Plan is the cheapest for values of g between the point of intersection of $y = M(g)$ and $y = B(g)$ and the point of intersection of $y = M(g)$ and $y = W(g)$. Solving for these points of intersection, we have

$$\begin{array}{rcl} M(g) = B(g) & & M(g) = W(g) \\ 30 + 10g = 15 + 20g & & 30 + 10g = 100 \\ -10g = -15 & & 10g = 70 \\ g = 1.5 & & g = 7 \end{array}$$

So, the Maize Plan is cheapest for $1.5 < g < 7$, i.e. it is cheapest if U.M. will use between 1.5 and 7 gigabytes of data per month.