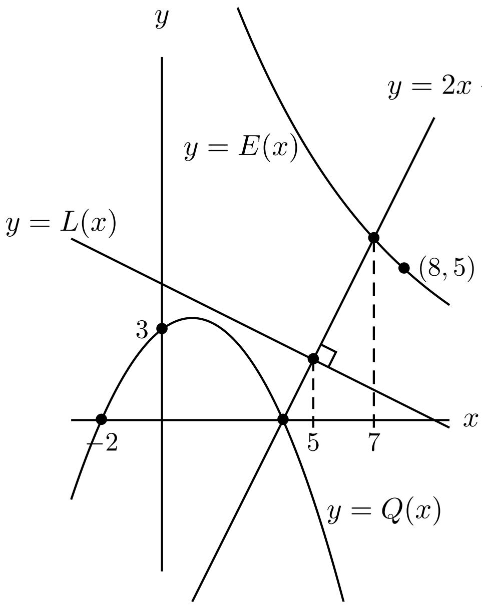
2. [10 points] The graph of $y = 2x - 8$ and of three functions L, Q, and E are shown below. Note that L is linear, Q is quadratic, and E is exponential.
Use the information shown in the graph to find formulas for $L(x)$, $Q(x)$, and $E(x)$.
Graphs may not be drawn to scale, so be careful! Use only the information that is labeled in the graph. Show your work clearly and leave all numbers in EXACT FORM.



Solution:
$\boxed{L(x)}$ First, note that the graph of $y = L(x)$ is perpendicular to the graph of $y = 2x - 8$, so the slope of the linear function L is $-1/2$. The y–coordinate of their point of intersection is $2(5) - 8 = 2$, so $(5, 2)$ is a point on the graph of $y = L(x)$. Using point-slope form, we have $L(x) - 2 = -\frac{1}{2}(x - 5)$ so $L(x) = 2 - \frac{1}{2}(x - 5) = 4.5 - \frac{1}{2}x$.

$\boxed{Q(x)}$ Solving $0 = 2x - 8$, we find that the x-intercept of $y = 2x - 8$ is 4. So the two zeros of Q are -2 and 4. Hence a formula for $Q(x)$ is $Q(x) = a(x + 2)(x - 4)$ for some constant a. Since the y-intercept is 3 we see that $3 = a(0 + 2)(0 - 4)$, so $3 = -8a$ and $a = -3/8$. Thus $Q(x) = -\frac{3}{8}(x + 2)(x - 4)$.

$\boxed{E(x)}$ The point of intersection of $y = E(x)$ and $y = 2x - 8$ is $(7, 6)$ (since $2(7) - 8 = 6$), so two points on the graph of $y = E(x)$ are $(7, 6)$ and $(8, 5)$. Since E is exponential and $8 - 7 = 1$, the growth/decay factor of E is $E(8)/E(7) = 5/6$. A formula for $E(x)$ is then $E(x) = c(5/6)^x$ for some constant c. Using the point $(7, 6)$ we find that $6 = c(5/6)^7$ so $c = 6(6/5)^7$. Thus $E(x) = 6(6/5)^7(5/6)^x$.

Remember: Show your work clearly in the space on this page, and leave all numbers in EXACT FORM. *Write your final answers in the answer blanks below.*

$L(x) =$ $4.5 - \frac{1}{2}x$

$Q(x) =$ $-\frac{3}{8}(x + 2)(x - 4) \left(= -\frac{3}{8}x^2 + \frac{3}{4}x + 3 \right)$

$E(x) =$ $\frac{6^8}{5^7}\left(\frac{5}{6}\right)^x$

3. [4 points] Find the average rate of change of the function $g(t) = 2t^2 - 3t + 4$ between $t = -1$ and $t = -1 + h$. *For full credit, simplify your answer as much as possible.*

Solution: The average rate of change is

$$\frac{g(-1+h) - g(-1)}{(-1+h) - (-1)} = \frac{\left(2(-1+h)^2 - 3(-1+h) + 4\right) - \left(2(-1)^2 - 3(-1) + 4\right)}{(-1+h) + 1}$$
$$= \frac{2(1 - 2h + h^2) - 3(-1+h) + 4 - (2 + 3 + 4)}{h}$$
$$= \frac{2h^2 - 7h}{h} = \frac{h(2h - 7)}{h} = \boxed{\text{2h-7}}.$$