5. [13 points] In 1940, there were 6.1 million farms in the United States, and this number decreased by a total of 60% during the next 40 years.

a. [2 points] Based on the data above, how many farms were there in the US in 1980?

Solution: In 1980, there were 40% as many farms as there were in 1940, so there were $0.4(6.1) = \boxed{2.44 \text{ million farms}}$.

b. [5 points] Suppose that the number of farms decreased *at a constant rate* from 1940–1980. Find a formula for $F(t)$, the number of millions of farms in the US this model predicts there were t years after 1940 .

Solution: Since the rate of change is constant, F is linear. The constant average rate of change (slope) of F is $\dfrac{F(40) - F(0)}{40 - 0} = \dfrac{2.44 - 6.1}{40} = -0.0915$ million farms per year. Since $F(0) = 6.1$, we use slope-intercept form to see that $\boxed{F(t) = 6.1 - 0.0915t}$.

According to this model, in what year were there (or will there be) a total of 4 million farms in the US?

Solution: We solve the equation $F(t) = 4$ and find

$$\begin{aligned} F(t) &= 4 \\ 6.1 - 0.0915t &= 4 \\ -0.0915t &= -2.1 \\ t &\approx 22.95 \end{aligned}$$

$\boxed{\text{So according to this model, there were 4 million farms in the US in about 1963.}}$

c. [6 points] Now, suppose instead that the number of farms decreased *at a constant percent rate* from 1940–1980. Under this new assumption, by what percent did the number of farms in the US decrease each year between 1940 and 1980?

Solution: Under this assumption, the number of farms is an exponential function of time. Let b be the annual decay factor. Then the number of farms in 1980 was $6.1(b^{40})$, so $2.44 = 6.1(b^{40})$. Thus $b^{40} = 0.4$ so $b = 0.4^{1/40} \approx 0.97735$. Hence the number of farms in the US decreased by $\boxed{\text{about } 2.23\% \text{ each year}}$ between 1940 and 1980.

Find a formula for $P(t)$, the number of millions of farms in the US this model predicts there were t years after 1940 .

Solution: This is the formula we were working with above. In particular, this is an exponential function with initial value 6.1. We found the annual decay factor b above, so we have $\boxed{P(t) = 6.1(0.4)^{t/40} \approx 6.1(0.9774)^t}$.