

6. [5 points] Let $f(x) = -4x^2 + 12kx - 17$. Use the method of completing the square to rewrite this function in vertex form and then give the coordinates of the vertex.
Show your work step-by-step. Note: Your answers may involve the constant k .

Solution:

$$\begin{aligned} f(x) &= -4x^2 + 12kx - 17 \\ &= -4(x^2 - 3kx) - 17 \\ &= -4 \left[x^2 - 3kx + \left(\frac{-3k}{2}\right)^2 - \left(\frac{-3k}{2}\right)^2 \right] - 17 \\ &= -4 \left[\left(x - \frac{3k}{2}\right)^2 - \frac{9k^2}{4} \right] - 17 = -4 \left(x - \frac{3k}{2}\right)^2 + 9k^2 - 17 \end{aligned}$$

Vertex form: $f(x) = -4 \left(x - \frac{3k}{2}\right)^2 + (9k^2 - 17)$

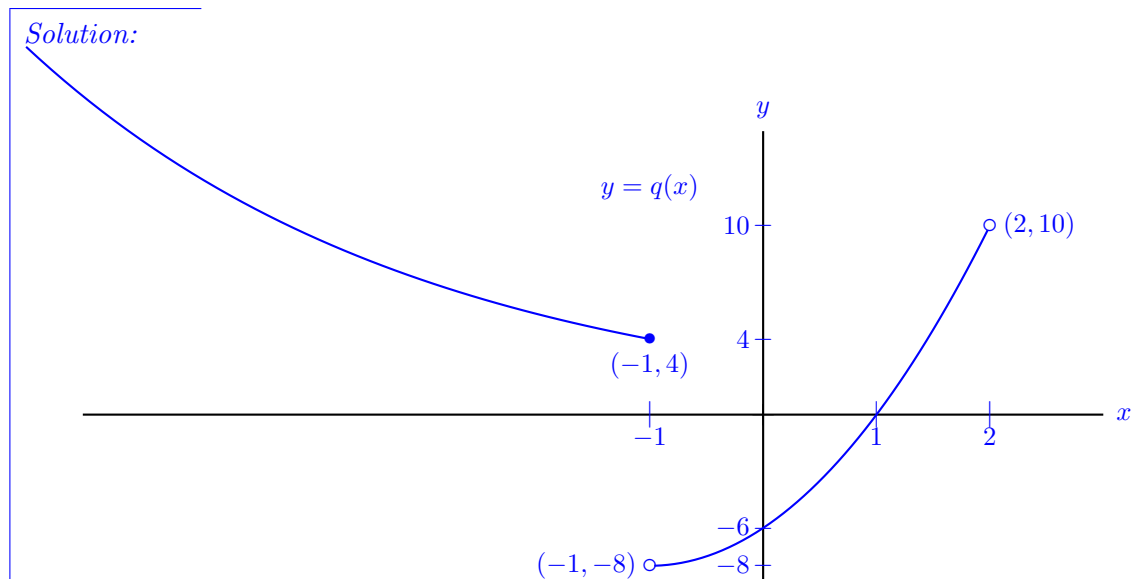
Vertex: $\left(\frac{3k}{2}, 9k^2 - 17\right)$

7. [10 points] Consider the function q defined by $q(x) = \begin{cases} 3(0.75)^x & \text{if } x \leq -1 \\ 2(x+1)^2 - 8 & \text{if } -1 < x < 2 \end{cases}$

- a. [2 points] Evaluate $q(q(0))$.

Solution: $q(q(0)) = q(2(0+1)^2 - 8) = q(2 - 8) = q(-6) = 3(0.75)^{-6}$.

- b. [4 points] Sketch a graph of $y = q(x)$. Carefully label your axes and important points on your graph (including intercepts).



- c. [4 points] Find the domain and range of q . (Use either interval notation or inequalities.)

Solution: Based on the given formula, we see that the domain of q is the interval $(-\infty, 2)$ and using the graph from part (b), we conclude that the range of q is the interval $(-8, \infty)$.

Domain: $(-\infty, 2)$

Range: $(-8, \infty)$