8. [15 points] The cost of computer memory has changed dramatically over time. Let $C(t)$ be the cost, in dollars per gigabyte, of computer memory t years after 1956. Some estimated data for C is provided in the table below.[1]

t	0	33	38	44	48	55
$C(t)$	10,000,000	36,000	1000	20	1	0.035

a. [3 points] Find and interpret, in the context of this problem, the average rate of change of $C(t)$ for $33 \leq t \leq 38$. (Use a complete sentence and include units.)

Solution: The average rate of change is $\frac{C(38)-C(33)}{38-33} = \frac{1000-36000}{5} = -7000$ dollars per gigabyte per year. So, between 1989 and 1994, the cost of computer memory decreased at an average rate of \$7000 per gigabyte per year.

Note: We collect the successive average rates of change of C for reference in parts (b)–(d) below.

interval	$0 \leq t \leq 33$	$33 \leq t \leq 38$	$38 \leq t \leq 44$	$44 \leq t \leq 48$	$48 \leq t \leq 55$
Δt (in years)	33	5	6	4	7
$\Delta C(t)$ (in \$/GB)	-9964000	-35000	-980	-19	-0.065
Avg rate of change (in \$/GB per yr)	≈ -301939.39	-7000	≈ -163.3	-4.75	≈ -0.00929

b. [4 points] Based on the data provided in the table above, could the function $C(t)$ be linear, exponential, or neither linear nor exponential? (*Circle one.*)

Linear   Exponential   **[Neither linear nor exponential]**

Justify your answer numerically (i.e. show your work and explain your reasoning).

Solution: The average rate of change is *not* constant (as can be seen in the table above), so the function is *not* linear.
Note that $C(44)/C(33) \approx 0.00056$ whereas $C(55)/C(44) = 0.00175$. Since the two time intervals $33 \leq t \leq 44$ and $44 \leq t \leq 55$ are both the same length (11 years), these ratios would be the same if $C(t)$ were exponential. Therefore $C(t)$ is *not* exponential.
(Note that alternatively, we could have computed the annual decay factor over each time interval in the table to see that this factor is not constant.)

c. [2 points] Based on the data provided in the table above, is the function $C(t)$ increasing, decreasing, or neither increasing nor decreasing on the entire interval from $t = 0$ to $t = 55$? (*Circle one.*)

Increasing   **[Decreasing]**   Neither increasing nor decreasing

Solution: The average rate of change over every time interval shown in the table is negative, so $C(t)$ appears to be decreasing over the entire interval from $t = 0$ to $t = 55$.

d. [2 points] Based on the data provided in the table above, is the function $C(t)$ concave up, concave down, or neither concave up nor concave down on the entire interval from $t = 0$ to $t = 55$? (*Circle one.*)

[Concave Up]   Concave Down   Neither concave up nor concave down

Solution: The average rate of change of $C(t)$ over successive time intervals is increasing (becoming "less negative"), so $C(t)$ appears to be concave up.

e. [4 points] Estimate $C^{-1}(46)$. Then interpret its meaning in the context of this problem. (Use a complete sentence and include units.)

Solution: $C^{-1}(46)$ is between 38 and 44, most likely closer to 44 (since 46 is much closer to 20 than to 1000). So, we estimate that $C^{-1}(46) \approx 43$.
This means that the cost of memory was 46 dollars per gigabyte in approximately 1999.

[1]Source: http://en.wikipedia.org/wiki/Memory_storage_density