9. [14 points] A fashion designer has a budget of \$300 for fabric for a fabulous garment. The designer is going to use a combination of denim fabric which costs \$8 per yard and jersey fabric which costs \$12 per yard. (Assume that the fabric store will sell any length of these fabrics, i.e. partial yards are okay.)
Assume that the designer spends the entire budget of \$300 on these two fabrics. Let D be the number of yards of denim and J be the number of yards of jersey that the designer purchases.

a. [2 points] In one complete sentence, explain why J is a function of D.

Solution: Each value of the input D determines exactly one value of the output J because once the designer decides on D, J is completely determined by the amount of money from the budget that is left over.

Let $f(D)$ be the number of yards of jersey that the designer buys if the designer buys D yards of denim, so $J = f(D)$.

b. [3 points] Evaluate $f(5)$ and interpret it in the context of this problem.
(Use a complete sentence and include units.)

Solution: $f(5)$ is the number of yards of jersey that the designer buys if he/she buys 5 yards of denim.
5 yards of denim costs a total of \$40, leaving \$260 for jersey. Each yard of jersey costs \$12, so the designer will buy $260/12 = 21\ 2/3$ yards of jersey. Hence $\boxed{f(5) = 21\ 2/3}$.
Interpretation: If the designer buys 5 yards of denim, then he/she buys $21\ 2/3$ yards of jersey.

c. [3 points] Find a formula for $f(D)$.

Solution: If the designer buys D yards of denim and J yards of jersey then he/she spends $\$8D$ on denim and $\$12J$ on jersey. Because the designer spends the entire budget on denim and jersey, we have $8D+12J = 300$ so solving for J we find $J = \frac{300-8D}{12} = 25-\frac{2}{3}D$.
Thus $\boxed{f(D) = 25 - \frac{2}{3}D}$.

d. [3 points] Find and interpret, in the context of this problem, the D–intercept of the graph of $J = f(D)$. (Use a complete sentence and include units.)

Solution: The D-intercept is the value of D when $J = 0$, which is the solution to $8D + 12(0) = 300$ or $\boxed{D = 37.5}$.
So, the designer buys 37.5 yards of denim if he/she buys no jersey.

e. [3 points] Give a practical interpretation of $f^{-1}(k)$ in the context of this problem.
(Use a complete sentence and include units. You do not need to find a formula.)

Solution: $f^{-1}(k)$ is the number of yards of denim the designer buys if he/she buys k yards of jersey. (Another phrasing: If the designer buys k yards of jersey, then he/she buys $f^{-1}(k)$ yards of denim.)