

3. [10 points] Annie Ant and Greta Grasshopper are having a debate about how to spend their time during October. Annie says that she will spend a total of 12 hours each day gathering food and building her anthill. Let B be the number of cm^3 of anthill that Annie builds in October, and let $D = g(B)$ be the number of grams of food that she gathers in October.

Annie knows that g is a linear function. She is also able to determine that if she builds 500 cm^3 of her anthill in October, then she will gather a total of 1500 grams of food but that if she builds only 150 cm^3 of her anthill, then she will gather a total of 2300 grams of food in October.

- a. [4 points] Find a formula for $g(B)$.

Solution: Since g is linear, we first find its constant average rate of change. We have $g(500) = 1500$ and $g(150) = 2300$, so the constant average rate of change (slope) of g is

$$\frac{g(500) - g(150)}{500 - 150} = \frac{1500 - 2300}{350} = \frac{-800}{350} = -\frac{16}{7}.$$

Using point-slope form, we find $g(B) - 1500 = -\frac{16}{7}(B - 500)$ so

$$g(B) = 1500 - \frac{16}{7}(B - 500) = \frac{18500}{7} - \frac{16}{7}B.$$

Answer: $g(B) = \underline{1500 - \frac{16}{7}(B - 500) = \frac{18500}{7} - \frac{16}{7}B}$

- b. [6 points] Find and interpret the slope and horizontal intercept of the graph of $D = g(B)$ in the context of this problem. For each interpretation, remember to use a complete sentence and include units.

Answers

Slope = $\underline{-\frac{16}{7} \text{ grams of food per cm}^3 \text{ of anthill}}$

Interpretation of slope: In October, for every 7 cm^3 of her anthill that Annie builds, she gathers 16 fewer grams of food.

Solution: The horizontal intercept occurs when $g(B) = 0$. Solving for B , we have $\frac{18500}{7} - \frac{16}{7}B = 0$ so $\frac{16}{7}B = \frac{18500}{7}$ and $B = \frac{18500}{16} = \frac{4625}{4} = 1156.25$.

Horizontal intercept = $\underline{\frac{18500}{16} = \frac{4625}{4} = 1156.25 \text{ cm}^3 \text{ of anthill}}$

Interpretation of horizontal intercept: In October, if Annie builds 1156.25 cm^3 of her anthill, she will gather no food.