5. [13 points] Roo is a boxing kangaroo in Australia. Every Sunday, Roo has a boxing match against a professional boxer at the Sydney Opera House.

Let $r(t)$ be the revenue, in dollars, that the opera house makes from ticket sales when it sells t tickets to one of Roo's matches. Then

$$r(t) = t\left(230 - \frac{1}{30}t\right).$$

Note: The capacity of the Sydney Opera House is 5738, so there are never more than 5738 tickets sold to a match.

a. [5 points] If the opera house had a revenue of \$159,120 from ticket sales to last week's match, how many tickets did they sell? *Remember to show your work carefully.*

Solution: If the opera house has a revenue of \$159,120, then $r(t) = 159120$. We use the quadratic formula to solve for t in this equation.

$$\begin{aligned} 159120 &= r(t) \\ 159120 &= t\left(230 - \frac{1}{30}t\right) \\ 159120 &= -\frac{1}{30}t^2 + 230t \\ 0 &= -\frac{1}{30}t^2 + 230t - 159120 \end{aligned}$$

$$\begin{aligned} \text{So } \ t &= \frac{-230 \pm \sqrt{230^2 - 4(-\frac{1}{30})(-159120)}}{2(-\frac{1}{30})} \\ &= \frac{-230 \pm \sqrt{52900 - 21216}}{-\frac{1}{15}} \\ &= -15(-230 \pm \sqrt{31684}) \\ &= -15(-230 \pm 178) = 780 \text{ or } 6120 \end{aligned}$$

Because the capacity of the opera house is 5738, the only valid solution is 780.

Answer: 780 **tickets**

b. [6 points] Use the method of completing the square to put the formula for $r(t)$ in vertex form. *Carefully show your algebraic work step-by-step.*

Solution:

$$\begin{aligned} r(t) &= t\left(230 - \frac{1}{30}t\right) = -\frac{1}{30}t^2 + 230t = -\frac{1}{30}(t^2 - 6900t) \\ &= -\frac{1}{30}\left[t^2 - 6900t + \left(\frac{-6900}{2}\right)^2 - \left(\frac{-6900}{2}\right)^2\right] \\ &= -\frac{1}{30}\left[(t-3450)^2 - (-3450)^2\right] \\ &= -\frac{1}{30}(t-3450)^2 + 396750 \end{aligned}$$

Answer: $r(t) =$ $-\frac{1}{30}(t-3450)^2 + 396750$

c. [2 points]

Solution: Using the vertex form we found above, the vertex is $(3450, 396750)$. Because the leading coefficient $(-\frac{1}{30})$ is negative, this gives the maximum value of the function. Thus the maximum revenue is \$396,750, and this occurs when 3450 tickets are sold.

What is the maximum possible revenue? $\$396,750$

How many tickets are sold to make the maximum possible revenue? 3450 **tickets**