8. [13 points] Roger the rabbit is a large rabbit that likes to eat! On a normal day, Roger has a daily meal of 12 ounces of carrots and 7 ounces of lettuce mixed together. However, sometimes Roger will want to eat a different mix for his daily meal. Let $R(z)$ be the ratio of the amount of lettuce in his food mix to the total amount of food if $|z|$ ounces of lettuce have been added ($z > 0$) or removed ($z < 0$). Note that Roger starts with 12 ounces of carrots and 7 ounces of lettuce and that the amount of carrots does NOT change.

a. [3 points] Evaluate $R(0)$, $R(4)$ and $R(-0.5)$.

Solution: $R(0)$ is the initial ratio of lettuce to food mix. So $R(0) = \frac{7}{7+12} = \frac{7}{19} \approx 0.3684$. $R(4)$ is the ratio of lettuce to food mix when he adds 4 ounces of lettuce to his food mix. Thus $R(4) = \frac{7+4}{7+12+4} = \frac{11}{23} \approx 0.47826$. Finally $R(-0.5)$ is the ratio of lettuce to food mix when he takes 0.5 ounces of lettuce out of his food mix. Hence $R(-0.5) = \frac{7-0.5}{7+12-0.5} = \frac{6.5}{18.5} = \frac{13}{37} \approx 0.35135$.

$R(0) =$ $\frac{7}{19}$ $R(4) =$ $\frac{11}{23}$ $R(-0.5) =$ $\frac{13}{37}$

b. [4 points] Find the domain and range of $R(z)$ in the context of this problem. *Use either inequalities or interval notation to express your answers.*

Solution: Roger cannot take more than 7 ounces of lettuce out of his mixture, but he can add as much lettuce as he wants to.[1] This means that the domain is $[-7, \infty)$. If he removes all 7 ounces of lettuce from his food mix, the ratio is 0. If he adds as much lettuce as possible, the ratio will be close to 1, but never attain this value. This means that the range is $[0, 1)$.[1]

Domain: $[-7, \infty)$ **Range:** $[0, 1)$

c. [2 points] Find a formula for $R(z)$ in terms of z.

Solution: When Roger adds z ounces of lettuce, there are $7 + z$ ounces of lettuce in the mixture and there are $12 + 7 + z$ total ounces of food mixture. Thus, the resulting ratio of the amount of lettuce in his food mix to the total amount of food mix is $R(z) = \dfrac{7+z}{12+7+z} = \dfrac{7+z}{19+z}$.

Answer: $R(z) =$ $\frac{7+z}{19+z}$

d. [4 points] If Roger wants a food mixture with 65% lettuce, how much lettuce must he add or remove to create this mixture? *Show your work carefully, round to the nearest* 0.1 *ounce, include units, and clearly indicate whether lettuce should be added or removed.*

Solution: We want to find z so that $R(z) = 0.65$, so we solve for z in the equation $\dfrac{7+z}{19+z} = 0.65$.

$$\frac{7+z}{19+z} = 0.65 \qquad z - 0.65z = 12.35 - 7$$
$$7 + z = 0.65(19 + z) \qquad 0.35z = 5.35$$
$$7 + z = 12.35 + 0.65z \qquad z = \frac{5.35}{0.35} = \frac{107}{7} \approx 15.2857$$

Answer: $\frac{107}{7}$ **or about 15.3 ounces of lettuce must be added**

[1] Note that there technically is a maximum amount of lettuce Roger could add due to the available supply of lettuce, so the domain is $[-7, L_{max}]$, where L_{max} is the maximum number of ounces of lettuce actually available to be added. If we consider this restriction on the lettuce supply, then the range is instead $\left[0, \frac{7+L_{max}}{19+L_{max}}\right]$.