

9. [10 points] Annie Ant finished building her anthill, and it immediately started eroding because of the weather. Every day, her anthill loses 1.5% of its volume. Let  $v(d)$  be the volume, in  $\text{cm}^3$ , of Annie's anthill  $d$  days after she finished building it. Assume that her anthill was  $1200 \text{ cm}^3$  when she finished building it.

- a. [2 points] Based on the description above, answer each of the following questions. In each case, *circle the one best answer*. Note: You do NOT need to explain your reasoning.

*Solution:* According to the description, the volume of the anthill changes at a constant percent rate per unit time, so  $v(d)$  is an exponential function. Since the anthill is losing volume,  $v(d)$  is a decreasing function. (The volume of the anthill is decaying exponentially.)

- (i) What kind of function is  $v(d)$ ?

linear     
  quadratic     
  exponential     
  NONE OF THESE

- (ii) Which of the following accurately describes  $v(d)$ ?

$v(d)$  is an increasing function.     
   $v(d)$  is a decreasing function.

NEITHER OF THESE

- b. [3 points] Find a formula for  $v(d)$  in terms of  $d$ .

*Solution:* The initial value is 1200. Because the anthill loses 1.5% of its volume every day, the decay factor of this exponential function is  $1 - 0.015 = 0.985$ . So a formula for the exponential function  $v(d)$  is

$$v(d) = 1200(0.985)^d$$

**Answer:**  $v(d) = \underline{\hspace{10em} 1200(0.985)^d \hspace{10em}}$

- c. [3 points] Give a practical interpretation of the expression  $v^{-1}(50)$  in the context of this problem. *Use a complete sentence and include units.* Note that you do not need to evaluate  $v^{-1}(50)$ .

*Solution:* After the anthill is finished being built, it takes  $v^{-1}(50)$  days for the volume of the anthill to be  $50 \text{ cm}^3$ .

- d. [2 points] Solve for  $a$  in the equation  $v^{-1}(a) = 10$ . *Either give your answer in exact form or rounded to the nearest 0.01.*

*Solution:* If  $v^{-1}(a) = 10$ , then  $v(10) = a$ . Therefore

$$a = v(10) = 1200(0.985)^{10} \approx 1031.67653071.$$

**Answer:**  $\underline{\hspace{10em} a = 1200(0.985)^{10} \quad \text{or} \quad a \approx 1031.68 \hspace{10em}}$