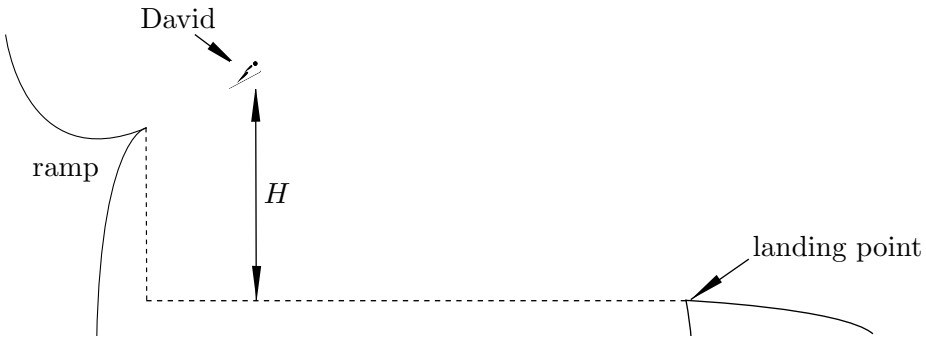
4. [16 points] David is a professional extreme athlete. In one of his stunts, he jumps off a ski ramp. David's height H (in m) above his landing point, from the moment he leaves the ramp until he lands, is given by the function

$$H = f(t) = -5t^2 + 8t + 15.$$

In this formula, t is the time (in seconds) after David leaves the ramp.



a. [3 points] Find the exact time it took David to travel from the ramp to his landing point? Include units.

Solution: The time it takes for David to reach his landing point has to satisfy $-5t^2 + 8t + 15 = 0$. Using the quadratic formula, we have

$$t = \frac{-8 \pm \sqrt{8^2 + 300}}{-10} = \frac{8 \pm \sqrt{364}}{10} = 0.8 \pm \sqrt{3.64}.$$

Thus, $t = 0.8 + \sqrt{3.64}$ seconds.

b. [5 points] Use the method of completing the square to write the formula for $f(t)$ in vertex form. Carefully show your algebraic work step-by step.

Solution:

$$\begin{aligned} f(t) = -5t^2 + 8t + 15 &= -5(t^2 - \frac{8}{5}t) + 15 \\ &= -5\left(t^2 - \frac{8}{5}t + \frac{16}{25} - \frac{16}{25}\right) + 15 \\ &= -5\left(t^2 - \frac{8}{5}t + \frac{16}{25}\right) + \frac{16}{5} + 15 \\ &= -5\left(t - \frac{4}{5}\right)^2 + \frac{91}{5} \end{aligned}$$

We rewrote the problem in this page for your convenience

David is a professional extreme athlete. In one of his stunts, he ski jumps off a ski ramp. David's height H (in m) above his landing point, from the moment he leaves the ramp until he lands, is given by the function

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In this formula, t is the time (in seconds) after David leaves the ramp.

c. [2 points] What is the exact value of David's maximum height above his landing point during his jump? Include units.

Solution: Since the vertex of the quadratic is at $\left(\frac{4}{5}, \frac{91}{5}\right)$, then the maximum height is at $\frac{91}{5}$ meters.
Answer=$\frac{91}{5}$ or 18.2 meters

d. [2 points] How high is the ramp above his landing point? Include units.

Solution: $f(0) = 15$ meters.
Answer:_____

e. [4 points] What is the domain and range of $H = f(t)$ in the context of this problem? Express your answer using inequalities or interval notation. Your answer has to be exact.

Solution: Domain:$[0, 0.8 + \sqrt{3.64}]$, Range:$[0, \frac{91}{5}]$