5. [10 points] While ski jumping, David broke his leg and was taken to the hospital. The hospital doctor administered a painkiller to David at noon. At 3 pm, the concentration of the painkiller in David's blood was 10 mg per liter and at 5 pm, it fell to 6 mg per liter. Let $C(t)$ be the concentration (in mg per liter) of the painkiller in David's blood t hours after noon. Suppose that the function C is decreasing exponentially.

a. [6 points] Find a formula for $C(t)$. Show all your work. Your answer must be exact.

Solution: Since C is an exponential function, $C(t) = ab^t$. Plug in $(3, 10)$ and $(5, 6)$ to obtain

$$10 = ab^3 \quad \text{and} \quad 6 = ab^5.$$

By taking the ratio, we get $b^2 = \frac{3}{5}$, so $b = \sqrt{0.6}$. Thus, $a = \frac{10}{\sqrt{0.6}^3}$, and so $C(t) = \frac{10}{\sqrt{0.6}^3}\left(\sqrt{0.6}\right)^t$.

b. [4 points] What is the hourly percentage growth rate of $C(t)$ and the initial concentration of painkiller in David's blood? Include units when appropriate. Your answer must be exact or accurate up to one decimal place.

Solution: Hourly percentage growth rate= -22.5%. Initial concentration= 21.5 mg/liter.

6. [5 points] For each of the following functions, write down its growth factor if the function is exponential or NONE if the function is not exponential.

(i) $f(t) = 2t^3$ Answer=_____

Solution: NONE

(ii) $g(t) = 2^t 3^t$ Answer=_____

Solution: 6

(iii) $h(t) = (3^{-t})^2$ Answer=_____

Solution: $\frac{1}{9}$