7. [12 points] Include all your work in the following problems to receive full credit.

a. [6 points] At the supermarket, you decide to buy blueberries and mangos. The price of blueberries is \$5.75 per pound and mangos cost \$3.20 per pound. Suppose that you spend \$30 buying B pounds of blueberries and M pounds of mangos. Let f be the function such that $B = f(M)$.

(i) Find a formula for f.

Solution: $30 = 5.75B + 3.20M \Rightarrow B = f(M) = \frac{30-3.20M}{5.75}$

(ii) Find the vertical intercept of the graph of the function f, and interpret this intercept using complete sentences. Include units, and your answer must be exact or accurate up to 2 decimal places.

Solution: Vertical intercept$= f(0) = \frac{30}{5.75} = 5.22$ pounds.
Practical interpretation: The vertical intercept is the number of pounds of blueberries I can buy if I spend all \$30 buying blueberries.

b. [6 points] A supermarket opens everyday at 8 am and closes at 6 pm. The supermarket manager notices that the amount of clients during a day is given by a quadratic function. Let $C(t)$ be the amount of clients in the supermarket t hours after the store opened. Find a formula for $C(t)$ if there are 250 clients in the store at 10 am, and there are no clients when the store opens and closes.

Solution: Since $C(0) = 0$ and $C(10) = 0$ and C is a quadratic function, we have that its factored formula is

$$C(t) = a(t-10)t$$

for some a. Plug in $(t, C) = (2, 250)$ to obtain $250 = -16a$ and solve for $a = -\frac{125}{8}$. Thus, $C(t) = -\frac{125}{8}t(t-10)$.

Solution: Let $C(t) = at^2 + bt + c$. Since $C(0) = 0$, we have $C(t) = at^2 + bt$ for some a, b. Plug in $(10, 0)$ to obtain $0 = 100a + 10b$ and plug in $(2, 250)$ to obtain $250 = 4a + 2b$. Hence

$$0 = 100a + 10b \qquad\qquad 250 = 4a + 2b.$$

Solving for a and b, you get $a = -\frac{125}{8}$ and $b = \frac{625}{4}$. Thus, $C(t) = -\frac{125}{8}t^2 + \frac{625}{4}t$.