8. [14 points] An ice cube is left to melt in a warm room. Let $V = f(t)$ be the volume of the ice cube (in cm^3) t seconds after it starts melting. Also, as the ice cube melts, a circular puddle of water of radius r (in cm) and area A (in cm^2) starts forming around it. Let g and h be functions such that $r = g(t)$ and $A = h(r)$. You may assume f, g and h are invertible.

a. [6 points] Select a mathematical expression from the list below that represents each of the following statements.

(i) When the volume of the ice cube is 30 cm^3, the radius of the water puddle around the ice cube is 6 cm.

Solution: E

Answer: _____

(ii) The radius of the water puddle grows by 6 cm between 20 and 30 seconds after the ice cube started melting.

Solution: G

Answer: _____

(iii) Between 20 and 30 seconds after the ice cube started melting, the radius of the water puddle grows, on average, by 6 cm per second.

Solution: B

Answer: _____

A) $f(g(6)) = 30$	**B)** $\dfrac{g(30) - g(20)}{10} = 6$	**C)** $g(30) = 6$
D) $\dfrac{g(30) - g(20)}{20} = 6$	**E)** $f(g^{-1}(6)) = 30$	**F)** $\dfrac{g(30) + g(20)}{2} = 6$
G) $g(30) - g(20) = 6$	**H)** $f(6) = 30$	**J)** $g(20) - g(30) = 6$

b. [4 points] The following statements are practical interpretations of mathematical expressions (not necessarily the ones listed above). Write the mathematical expression in each case.

(i) The time (in seconds) it takes for the radius of the water puddle around the ice cube to be 7 cm.

Solution: $g^{-1}(7)$

Answer: _____

(ii) The area (in cm^2) of the circular water puddle formed around the ice cube 9 seconds after the ice cube started melting.

Solution: $h(g(9))$

Answer: _____

c. [4 points] Assume that the domains of f and g is the interval of time it takes for the entire ice cube to melt. Indicate if the following functions are increasing, decreasing or neither.

Solution: $f(t)$ represents the volume of the ice cube (in cm^3) at time t (in seconds). Since the ice cube is melting, then $f(t)$ is decreasing. $h(g(t))$ represents the area of the water puddle forming around the ice cube (in cm^2) at time t (in seconds). Since the ice cube is melting, the area of the water puddle is increasing.

$f(t)$ is decreasing and $h(g(t))$ is increasing.